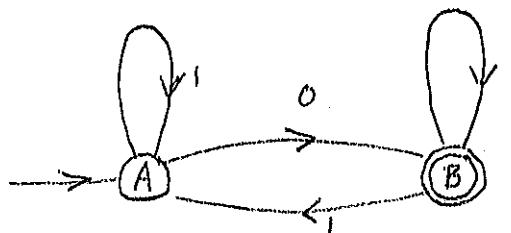


Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. Begin each of the 6 questions on a separate page.

- (15) 1. Let L be the language accepted by the NFA shown on the right.
Find NFAs which accept
(a) L^c (b) $(L^c)^R$.

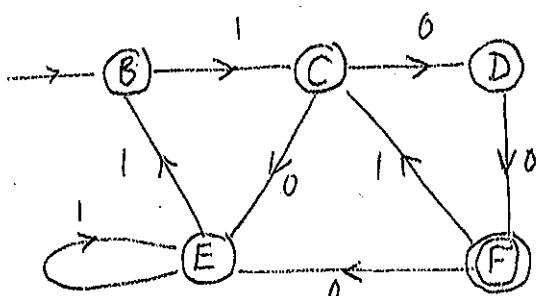


- (15) 2 (a) Find an NFA which is equivalent to the RLG given below.

$$G: \quad \begin{matrix} \rightarrow E, & E \rightarrow 0E, & E \rightarrow 1A, & A \rightarrow 1B, & A \rightarrow \lambda, & B \rightarrow 01, \\ B \rightarrow 1C, & C \rightarrow E, & C \rightarrow 0D, & D \rightarrow 10D, & D \rightarrow 01, & D \rightarrow \lambda. \end{matrix}$$

- (b) Suppose F is finite and $L-F$ is a regular language. Is it always true that L must also be a regular language? (Justify your answer.)

- (15) 3 (a) Write down what the Pumping Lemma says.
(b) Find a regular expression for the language accepted by the NFA shown on the right.

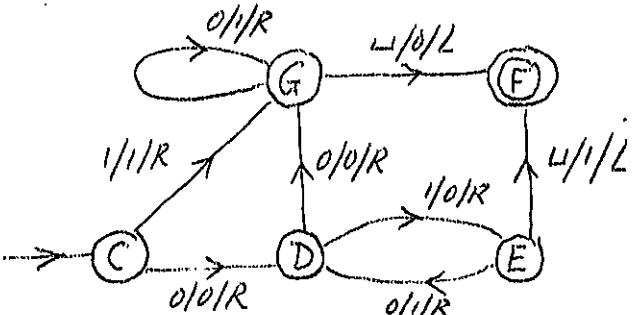


- (22) 4 (a) Define what it means for f to be obtained from g and h by primitive recursion. Show that $f(x,y) = 3x+y+2$ is a primitive recursive function by finding primitive recursive functions g and h such that $f = \text{prim.rec.}(g,h)$.

- (b) Define what it means for f to be obtained from g by minimization. Show that $f(x) = \sqrt{2x+1}$ is a recursive function by finding a primitive recursive function g such that $f = \mu[g, 0]$.

[You may use the fact that MONUS, ADD, & MULT are prim. rec. if needed in 4(b), but you are not allowed to do so in 4(a).]

- (15) 5 (a) Define what is a Turing-semi-decidable relation on N .
(b) Show what happens at each step if 0100 is the input for the TM, M shown on the right.
(c) Find the language accepted by M .



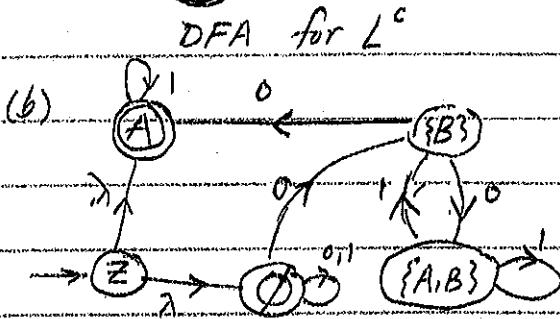
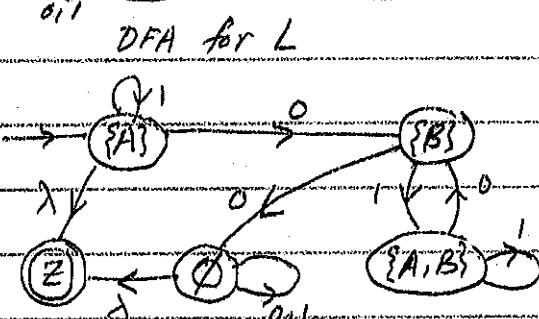
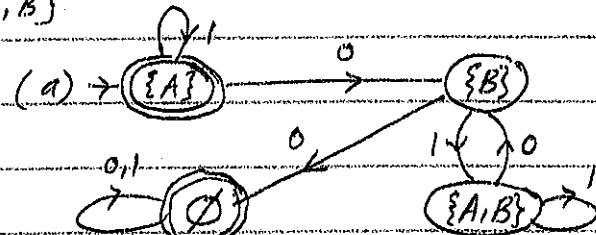
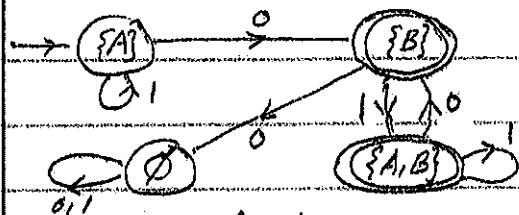
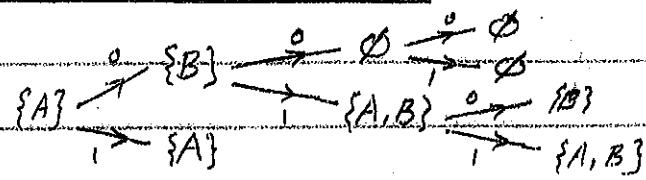
- (18) 6. Determine which of the following languages are regular and which are not. (a) $L_1 = \{a^k.b^n : k \pmod 3 < n^2 \pmod 3\}$ (b) $L_2 = \{c^k.d^n : k < n^2\}$.

[If you say that it is regular, you must find a regular expression; if you say it is non-regular, you must give a complete proof.]

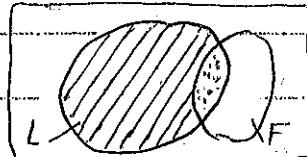
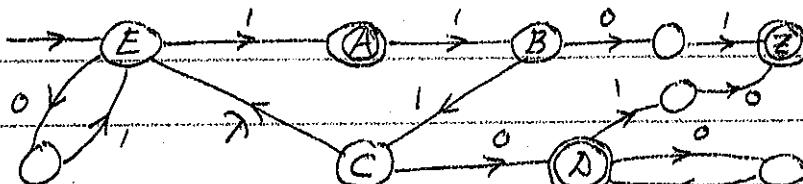
Solutions to Test #2

Fall 2012.

1.



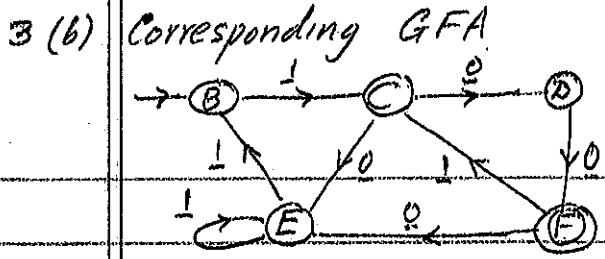
2(a)



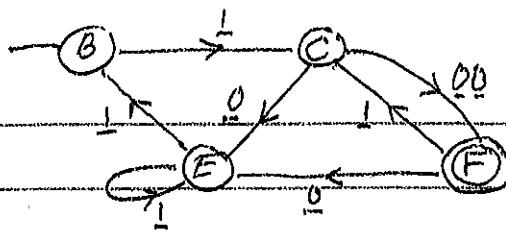
(b) YES. First observe that since F is finite, $F \cap L$ will also be finite. Since finite languages are regular, it follows that $F \cap L$ will be regular. Now $L = (L - F) \cup (F \cap L)$ and since $L - F$ is given as regular, it follows by the closure theorem that $(L - F) \cup (F \cap L) = L$ will be regular.

3(a)

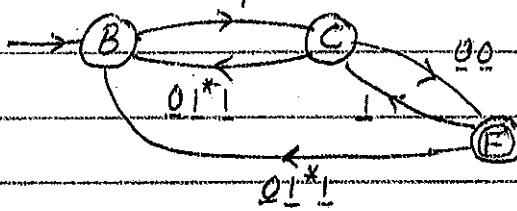
Pumping Lemma: Suppose L is an infinite regular language. Then we can find an $m \in \mathbb{N}$ such that any string $w \in L$ with $|w| \geq m$ can be decomposed as $w = xyz$ such that $|xy| \leq m$, $|y| \geq 1$, and $xy^i z \in L$ for each $i \in \mathbb{N}$.



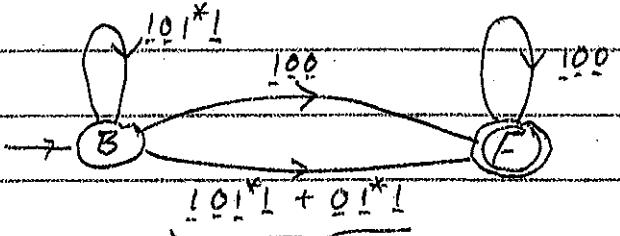
1. Eliminate D



2. Eliminate E



3. Eliminate C



$$\begin{aligned}
 L(E) &= r_1^* r_2^* (r_4 + r_3 r_1^* r_2)^* \\
 &= (101^*)^* \cdot 100 \cdot (100 + (1+\lambda) 01^* 1 \cdot (101^*)^* \cdot 100)^*.
 \end{aligned}
 = (1+\lambda) \cdot 01^* 1$$

4(a) f' is obtained from g & h by primitive recursion if
 $f(x, 0) = g(x)$ and $f(x, y+1) = h(x, y, f(x, y))$. Here
 $x = \langle x_1, \dots, x_n \rangle$, $g: \mathbb{N}^n \rightarrow \mathbb{N}$, $h: \mathbb{N}^{n+2} \rightarrow \mathbb{N}$ & $f: \mathbb{N}^n \rightarrow \mathbb{N}$.

$$f(x, y) = 3x + y + 2 \quad \text{So } f(x, 0) = 3x + 2 \Rightarrow g(x) = 3x + 2.$$

$$\text{Also } f(x, y+1) = 3x + (y+1) + 2 = (3x + y + 2) + 1 = f(x, y) + 1$$

$$\Rightarrow h(x, y, f(x, y)) = f(x, y) + 1. \quad \therefore h = s_0 s_0 I_3^{(3)}.$$

$$\text{Now } g(0) = 2 = s_0 s_0 0 \quad \text{and } g(y+1) = 3(y+1) + 2 = (3y+2) + 3 = g(y) + 3$$

$$\therefore g = \text{prim. rec. } (s_0 s_0 0, s_0 s_0 s_0 I_2^{(2)}). \quad \text{So}$$

$$f = \text{prim. rec. } (g, h) = \text{prim. rec. } (\text{prim. rec. } (s_0 s_0 0, s_0 s_0 s_0 I_2^{(2)}, s_0 I_3^{(3)}))$$

and hence f is a primitive recursive function

(b) f is obtained from g by minimization if $f: \mathbb{N}^n \rightarrow \mathbb{N}$
& $f(x) = \begin{cases} \text{smallest } y \text{ such that } g(x, y) = 0 \\ \text{undefined} & \text{if } g(x, y) \neq 0 \text{ for every } y. \end{cases}$

Here $x = \langle x_1, \dots, x_n \rangle$ & $g: \mathbb{N}^{n+1} \rightarrow \mathbb{N}$ is a total function.

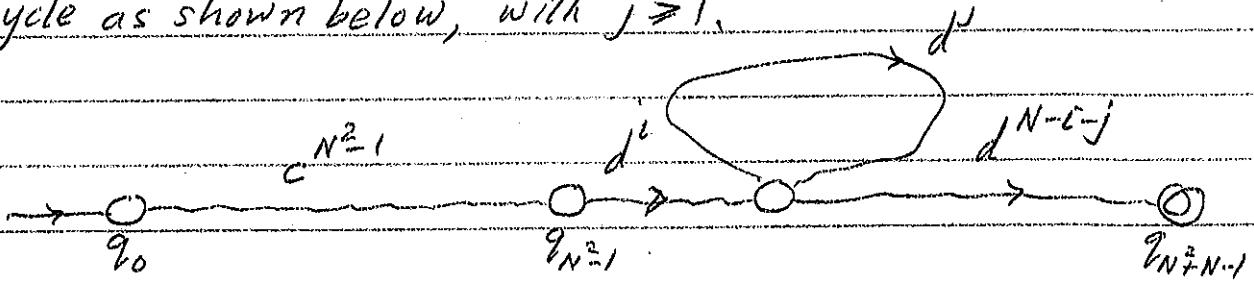
4(b) Let $g(x, y) = 2x+1 - y^2$. Then $(\mu y)[g(x, y) = 0]$
 $= (\mu y)[2x+1 - y^2 = 0] = \sqrt{2x+1} = f(x)$. $\therefore f = \mu[g, 0] =$
 $\mu[\text{MONUS} \circ [\text{SO ADD} \circ [I_1^{(2)}, I_1^{(2)}], \text{MULT} \circ [I_2^{(2)}, I_2^{(2)}], 0]]$
and so f is a recursive function

5(a) The relation R is Turing semi-decidable if we can find a TM M which halts in an accepting state whenever $\langle m, n \rangle \in R$; and which fails to halt or halts in a non-acc. state when $\langle m, n \rangle \notin R$.

(b) $\langle C, 0100 \rangle \vdash \langle D, 0100 \rangle \vdash \langle E, 0000 \rangle \vdash \langle F, 0010 \rangle \vdash \langle G, 0010 \rangle \vdash \langle H, 0010 \rangle$
(c) $L(M) = L_1 + L_2 + L_3$, where $L_1 = 1.0^* + 0.(0)^* 0.0^* + 0.1.(01)^*$.

6(a) $n \equiv 0 \pmod{3} \Rightarrow k \pmod{3} < 0$ (not poss.), $n \equiv 1 \pmod{3} \Rightarrow k \pmod{3} < 1^2$
 $n \equiv 2 \pmod{3} \Rightarrow k \pmod{3} < 2^2 \pmod{3} \Rightarrow k \equiv 0 \pmod{3} \Rightarrow k = 0 \pmod{3}$
 $\therefore L_1 = (aaa)^*(bbb)^* b + (aaa)^*(bbb)^* bb$ is a regular lang.

(b) $L_2 = \{c^k d^n : k < n^2\}$. Supp. L_2 was regular. Then we can find an NFA M such that $L(M) = L_2$. Let N = the number of states in M . Then $c^{N^2-1} d^N \in L_2$ because if we put $k = N^2-1$ & $n = N$ we will see that $k < n^2$. So M will accept $c^{N^2-1} d^N$. Since it takes $N+1$ states to process the d^N , the acceptance track of $c^{N^2-1} d^N$ must have a cycle as shown below, with $j \geq 1$.



Now if we skip the cycle, we will see that M accepts the string $c^{N^2-1} d^i d^{N-i-j} = c^{N^2-1} d^{N-j}$. But $c^{N^2-1} d^{N-j}$ is not in L_2 because $N^2-1 \neq (N-j)^2$. So this contradicts the fact that $L(M) = L_2$. Hence L_2 is non-regular.