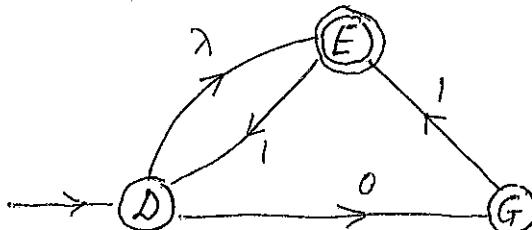


Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

- (15) 1. (a) Define what is the *transition relation* Δ of an NFA.

- (b) Convert the NFA on the right into an *equivalent DFA*.



- (15) 2. Find *regular expressions* which describe the languages below.

- (a) $L_1 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains both } 110 \text{ and } 011 \text{ as substrings}\}$
 (b) $L_2 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains at most one occurrence of the string } bb\}$

- (20) 3. (a) Define what it means for H to be an *inaccessible state* of a DFA M.

- (b) Partition the states of the DFA below into *blocks of indistinguishable states* & then find the *reduced machine*.

	A	B	\rightarrow	C	D	E	F	G
0	E	G		B	C	B	B	A
1	D	B		D	A	A	G	F

- (15) 4. (a) Let $f(\varphi) = [2n_a(\varphi) - 3 \cdot \{n_b(\varphi)+1\}] \pmod{4}$. Find a DFA which accepts the language $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is even}\}$.

- (b) If $\varphi = ababa$ find $f(\varphi)$ and then check your DFA with *ababa* as input.

- (20) 5. (a) Find a *context-free grammar* which generates the language

$$L_5 = \{a^n b^k : n > 3k + 1\} \cup \{b^n c^k : k < 2n+4\}.$$

- (b) Using your CFG, find *derivations* for each of the strings $a^5 b^1$ and $b^2 c^4$.

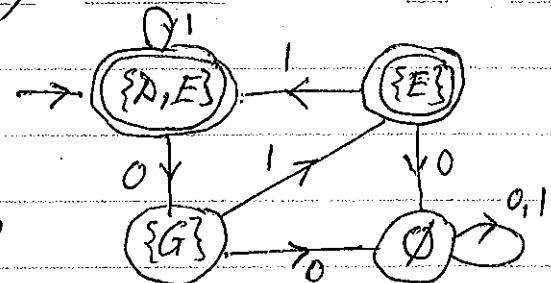
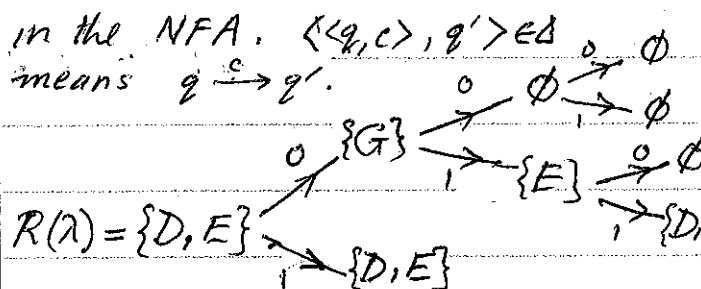
- (15) 6. Let A, B, and C be languages based on the *alphabet* $\{0,1\}$.

- (a) Is it always true that $(A - B) \cdot C \subseteq (A \cdot C) - (B \cdot C)$?

- (b) Is it always true that $(A \cap B) \cdot C \subseteq (A \cdot C) \cap (B \cdot C)$?
Justify your answers completely.

- 1(a) The transition relation of an NFA is a binary relation Δ from $Q \times (T \cup \{\lambda\})$ to Q , which gives us all the transitions in the NFA. $\langle \langle q, c \rangle, q' \rangle \in \Delta$ means $q \xrightarrow{c} q'$.

(b)



2(a)

$$\dots 110 \dots 011 \dots, \dots 011 \dots 110 \dots, \dots 11011 \dots, \dots 0110 \dots, \dots 01110 \dots$$

$$(0+1)^* (110 \cdot (0+1)^* 011 + 011 \cdot (0+1)^* 110 + 11011 + 0110 + 01110) \cdot (0+1)^*$$

(b)

no bb's; bb, one bb: bb \dots

$$(a+b\bar{a})^* \cdot (\underline{a} + \underline{b}) + (a+b\bar{a})^* \cdot \underline{bb} \cdot (\underline{a} + \underline{ab})^*$$

3(a)

The state H of a DFA M is inaccessible if there is no string $\varphi \in T^*$ such that $\delta^*(q_0, \varphi) = H$.

(b)

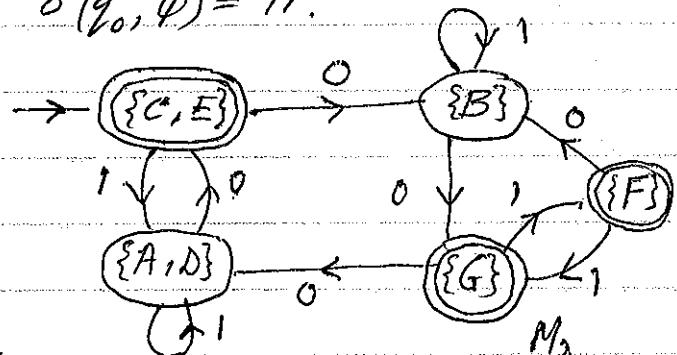
$$P_0 : \{A, B, D\} \quad \{C, E, F, G\}$$

$$P_1 : \{A, B, D\} \quad \{C, E\} \quad \{F, G\}$$

$$P_2 : \{A, D\} \quad \{B\} \quad \{C, E\} \quad \{F, G\}$$

$$P_3 : \{A, D\} \quad \{B\} \quad \{C, E\} \quad \{F\} \quad \{G\}$$

$$P_4 : \{A, D\} \quad \{B\} \quad \{C, E\} \quad \{F\} \quad \{G\} = P_3.$$



4(a)

Let A_i ($i = 0, 1, 2, 3$) keep track of the fact that $f(\varphi) \equiv i \pmod{4}$.

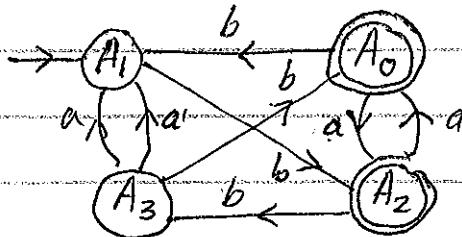
Since $f(\lambda) = 2(0) - 3(0+1) = -3 \equiv 1 \pmod{4}$, A_1 will be the initial state. The accepting states will be A_0 & A_2 because 0 & 2 are even. Now

$$\begin{aligned} f(\varphi a) &= 2n_a(\varphi a) - 3\{n_b(\varphi a) + 1\} = 2[n_a(\varphi) + 1] - 3\{n_b(\varphi) + 1\} \\ &= 2n_a(\varphi) - 3\{n_b(\varphi) + 1\} + 2 = f(\varphi) + 2 \pmod{4}. \end{aligned}$$

4(a) Also $f(\varphi b) = 2n_a(\varphi b) - 3\{n_b(\varphi b) + 1\} = 2n_a(\varphi) - 3\{n_b(\varphi) + 1 + 1\}$
 $= 2n_a(\varphi) - 3\{n_b(\varphi) + 1\} - 3 = f(\varphi) - 3 \equiv f(\varphi) + 1 \pmod{4}$

(b) $f(ababa) = 2(3) - 3(2+1)$
 $= -3 \equiv 1 \pmod{4}$

Input: $a \cdot b \ a \ b \ a$
states $A_1 \ A_3 \ A_0 \ A_2 \ A_3 \ A_1$



5(a) $S \rightarrow A/B$

This gives the union

$A \rightarrow aaaAb/C, C \rightarrow aC/aa$ This gives $\{a^n b^k : n \geq 3k+2\}$

$B \rightarrow bBDD/DDD, D \rightarrow c/\lambda$ This gives $\{b^n c^k : k \leq 2n+3\}$

(b) $S \Rightarrow A \Rightarrow aaaAb \Rightarrow aaaCb \Rightarrow aaaaab = a^5 b^1$

$S \Rightarrow B \Rightarrow bBDD \Rightarrow bbBD^4 \Rightarrow bbD^2 DDD^4 \Rightarrow bbA.BD.D^4$

$\Rightarrow bbA.A.D.D^4 \Rightarrow bb.A.A.A.D^4 \Rightarrow bbccD^3 \Rightarrow bbccD^2 \Rightarrow b^2 c^3 D \Rightarrow b^2 c^4.$

6(a) NO. Let $A = \{0\}$, $B = \{01\}$ and $C = \{1, 11\}$. Then

$$(A-B).C = (\{0\} - \{01\}).\{1, 11\} = \{0\}.\{1, 11\} = \{01, 011\}.$$

$$A.C - B.C = \{0\}.\{1, 11\} - \{01\}.\{1, 11\} = \{01, 011\} - \{011, 0111\} = \{01\}$$

So it is not always true that $(A-B).C \subseteq A.C - B.C$.

(b) YES. Let $\varphi \in (A \cap B).C$. Then $\varphi = \alpha \cdot \gamma$ with $\alpha \in A \cap B$ and $\gamma \in C$. Since $\alpha \in A \cap B$, $\alpha \in A$ and $\alpha \in B$.

So $\varphi = \alpha \cdot \gamma \in A.C$ because $\alpha \in A$ & $\gamma \in C$

Also $\varphi = \alpha \cdot \gamma \in B.C$ because $\alpha \in B$ & $\gamma \in C$.

Hence $\varphi \in (A.C) \cap (B.C)$ Thus $\varphi \in (A \cap B).C \subseteq (A.C) \cap (B.C)$.

END.