

MAD 3512 - THEORY OF ALGORITHMS

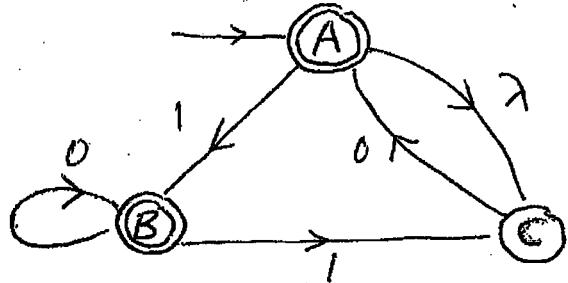
TEST #1 - Fall 2019

FLORIDA INT'L UNIV.

TIME: 75 min.

Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit.
BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1.(a) Define what is a *regular expression* over the alphabet $V = \{1, 2, 3\}$.
 (b) Let M be the NFA on the right. Find a DFA, M_D which is *equivalent* to M .



- (15) 2. Find *regular expressions* which describe the languages below.
 (a) $L_1 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains both } 110 \text{ and } 010 \text{ as substrings}\}$
 (b) $L_2 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains at most one occurrence of the string } aa\}$.

- (20) 3. (a) Define what it means for a state q to be *inaccessible* in a DFA, M .
 (b) Check for inaccessible states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent *reduced machine*, M_R .

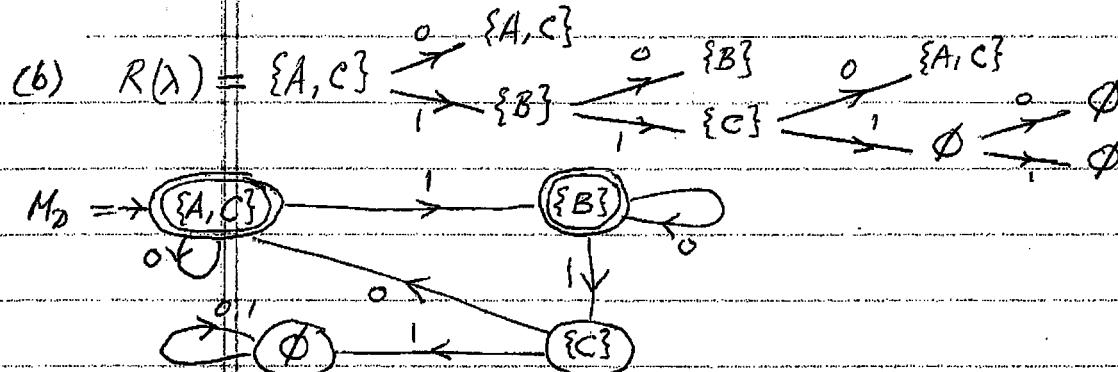
	A	B	C	D	E	F	G
0	F	F	C	B	G	B	E
1	C	D	G	C	C	A	F

- (15) 4. (a) Let $f(\omega) = [2.n_a(\omega) - n_b(\omega) - 3] \pmod{4}$. Find a DFA, M which accepts the language, $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is } 0 \text{ or } 2 \pmod{4}\}$.
 (b) If $\varphi = babab$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

- (20) 5. (a) Find a *context-free grammar* G which generates the language
 $L_5 = \{a^k b^n : n \geq 2k+3, k \geq 0\} \cup \{b^k c^n : 0 \leq n \leq 3k+2, k \geq 0\}$.
 (b) Find *derivations* in G for each of the strings: (i) $a^1 b^6$ and (ii) $b^1 c^4$.

- (15) 6. Let A , B , and C be languages based on the *alphabet* $\{a,b\}$.
 (a) Is it always true that $(A.C) \cup (B.C) \subseteq (A \cup B).C$?
 (b) Is it always true that $(A - B).C \subseteq (A.C) - (B.C)$? (Justify your answers.)

- 1(a) A regular expression over $V = \{1, 2, 3\}$ is defined recursively as follows:
- (i) $1, 2, 3, \lambda$, and \emptyset regular expressions; and
 - (ii) if E & F are regular expressions, then so are $(E+F)$, $(E \cdot F)$, & (E^*) .



2(a) $--110--010--$, $--010--110--$, $--\overline{11010}--$

$$E_1 = (\underline{0+1})^* (110 \cdot (\underline{0+1})^* \cdot 010 + 010 (\underline{0+1})^* \cdot 110 + 11010) \cdot (\underline{0+1})^*$$

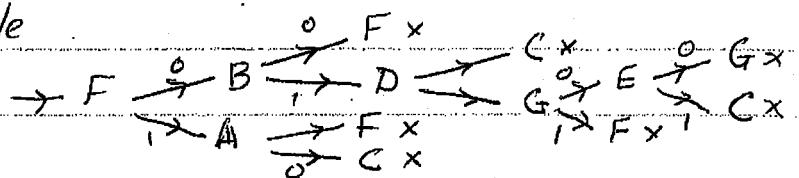
(b) $--\dots$ no aa's $--\dots$ no aa's $--\dots$ aa $--\dots$ no aa's

$$E_2 = (b+ab)^* + (b+ab)^* \cdot a + (b+ab)^* \cdot aa \cdot (b+ba)^*$$

- 3(a) A state q in a DFA M is inaccessible if there is no string $\varphi \in T(M)^*$ such that $\delta^*(q_0, \varphi) = q$.

(b) Let's check for inaccessible states in M .

So there are none in M .



(c) $P_0: \{A, D, E, G\} \quad \{B, C, F\}$

$P_1: \{A, D\} \quad \{E, G\} \quad \{B, C, F\}$

$P_2: \{A, D\} \quad \{E, G\} \quad \{B, F\} \quad \{C\}$

$P_3: \{A, D\} \quad \{E\} \quad \{G\} \quad \{B, F\} \quad \{C\}$

$P_4: \{A, D\} \quad \{E\} \quad \{G\} \quad \{B, F\} \quad \{C\} = P_3$



0	$\{B, F\}$	$\{C\}$	$\{B, F\}$	$\{G\}$	$\{E\}$
1	$\{A, D\}$	$\{G\}$	$\{C\}$	$\{C\}$	$\{B, F\}$

Let A_i ($i = 0, 1, 2, 3$) keep track that $f(\varphi) \equiv i \pmod{4}$.

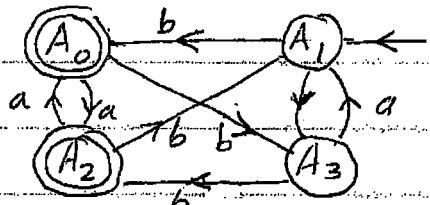
4(a) $f(\lambda) = 2n_a(\lambda) - n_b(\lambda) - 3 = 2(0) - 0 - 3 = -3 \equiv 1 \pmod{4}$, so A_1 = init state.

A_0 and A_2 are the accepting states because $\varphi \in L_4 \Leftrightarrow f(\varphi) \equiv 0 \text{ or } 2 \pmod{4}$.

$$f(wa) = 2n_a(wa) - n_b(wa) - 3 = 2n_a(w) - n_b(w) - 3 + 2 = f(w) + 2 \pmod{4}$$

$$f(wb) = 2n_a(wb) - n_b(wb) - 3 = 2n_a(w) - n_b(w) - 3 - 1 \equiv f(w) + 3 \pmod{4}.$$

So $M =$



$b \ a \ b \ a \ b \ \square$

$A_1 \ A_0 \ A_2 \ A_1 \ A_3 \ A_2 \checkmark$

$$f(babab) = 2(2) - 3 - 3 = -2 \equiv 2 \pmod{4} \checkmark$$

5(a) $\rightarrow S, S \rightarrow A/B, A \rightarrow aAbb/E, E \rightarrow Ebb/bbb,$
 $B \rightarrow bBCCC/CC, C \rightarrow c/\lambda$.

(b) $S \rightarrow A \Rightarrow aAbb \Rightarrow aEbb \Rightarrow aEbb \Rightarrow abbbb = a'b^6 \checkmark$

$$S \Rightarrow B \Rightarrow bBCCC \Rightarrow bCC.CCC \Rightarrow b\lambda C.CCC \Rightarrow b\lambda cCCC \Rightarrow b\lambda c.cCC \\ \Rightarrow b\lambda c.cC \Rightarrow b\lambda cccc = b'c^4 \checkmark$$

6(a) YES. Let $\varphi \in (A \cdot C) \cup (B \cdot C)$. Then $\varphi = A \cdot C$ or $\varphi \in B \cdot C$.

Case(i): $\varphi \in A \cdot C$: In this case $\varphi = \alpha \cdot \gamma$ for some $\alpha \in A$ & $\gamma \in C$.

Since $A \subseteq A \cup B$, $\alpha \in A \cup B$. $\therefore \varphi = \alpha \cdot \gamma$ with $\alpha \in A \cup B$ & $\gamma \in C$
 $\therefore \varphi \in (A \cup B) \cdot C$

Case(ii): $\varphi \in B \cdot C$: In this case $\varphi = \beta \cdot \gamma$ for some $\beta \in B$ & $\gamma \in C$.

Since $B \subseteq A \cup B$, $\beta \in A \cup B$. $\therefore \varphi = \beta \cdot \gamma$ with $\beta \in A \cup B$ & $\gamma \in C$
 $\therefore \varphi \in (A \cup B) \cdot C$

So in either case $\varphi \in (A \cup B) \cdot C$. Hence $(A \cdot C) \cup (B \cdot C) \subseteq (A \cup B) \cdot C$.

(b) NO. Let $A = \{a\}$, $B = \{ab\}$, and $C = \{a, ba\}$. Then

$$(A - B) \cdot C = (\{a\} - \{ab\}) \cdot \{a, ba\} = \{a\} \cdot \{a, ba\} = \{aa, aba\},$$

$$A \cdot C = \{a\} \cdot \{a, ba\} = \{aa, aba\} \text{ & } B \cdot C = \{ab\} \cdot \{a, ba\} = \{aba, abba\}.$$

$$\text{So } (A \cdot C) - (B \cdot C) = \{aa, aba\} - \{aba, abba\} = \{aa\}. \text{ Hence}$$

$(A - B) \cdot C = \{aa, aba\} \neq \{aa\} = (A \cdot B) - (B \cdot C)$. Thus it is not always true that $(A - B) \cdot C \subseteq (A \cdot B) - (B \cdot C)$. END