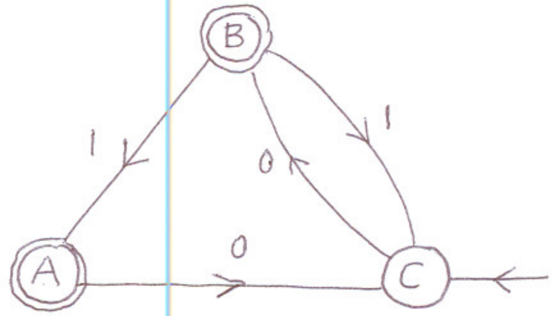
MAD 3512 - THEORY OF ALGORITHMS FLORIDA INT'L UNIV.

TEST #1 - SPRING '08 TIME: 75 min.

Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

(15) 1. (a) Define what is the *extended transition function* of a DFA.

(b) Convert the NFA on the right into an equivalent DFA.



(15) 2. Find regular expressions which describe the languages below

(a) $L_1 = \{\alpha \in \{a,b\}^* : \alpha$ contains both aab & bab as substrings$\}$

(b) $L_2 = \{\beta \in \{0,1\}^* : \beta$ has at most one occurrence of 01$\}$

(20) 3. (a) Find all the inaccessible states in the DFA below.

(b) Then partition the remaining states into blocks of indistinguishable states and find the reduced machine.

	A	B	C	→D	E	F	G	H
0	D	G	B	B	G	C	A	B
1	F	B	F	A	E	A	H	G

(15) 4. (a) Find a DFA which accepts precisely the strings in the language $L_4 = \{\omega \in \{a,b\}^* : [n_b(\omega) - n_a(\omega) - 1] \pmod 4 < 2\}$.

(b) Verify your DFA works with babb as input.

(20) 5. (a) Define what are *unreachable productions and non-terminating productions* in a CFG G.

(b) Find a context-free grammar which generates the language $L_5 = \{a^k b^n : k < n+3\} \cup \{b^k a^n : k > 2n+2\}$.

(15) 6. Let A, B and C be languages based on the alphabet $\{0,1\}$.

(a) Is it always true that $(B.A^R) \cup (C.A^R) \subseteq (B \cup C).A^R$?

(b) Is it always true that $(B.A^R) \cap (C.A^R) \subseteq (B \cap C).A^R$?

Justify your answers completely.

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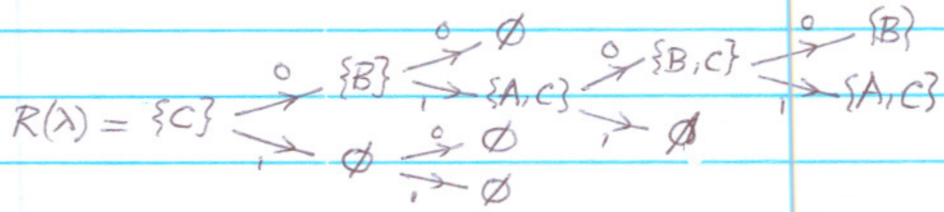
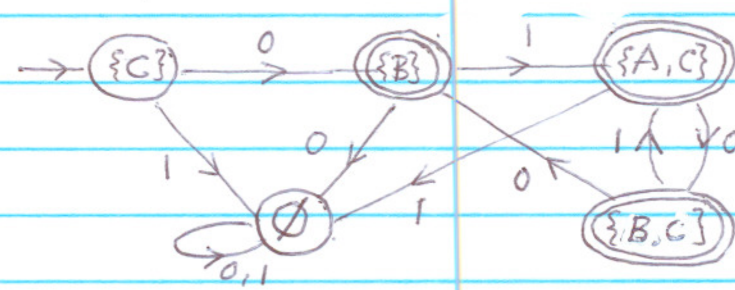
Solutions to Test #1 — Spring 2008

1(a) The extended transition function of a DFA is defined recursively by $\delta^*(q,\lambda)=q$ and $\delta^*(q,\varphi a)=\delta(\delta^*(q,\varphi),a)$ for any $a\in\Sigma$.

(b)



The DFA is



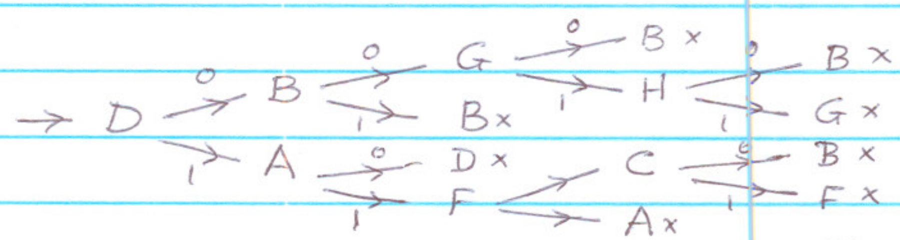
2(a) ....aab...bab... , ...bab...aab... , ...aabab...

$E_1 = (\underline{a}+\underline{b})^*.(\underline{aab}.(\underline{a}+\underline{b})^*.\underline{bab} + \underline{bab}.(\underline{a}+\underline{b})^*.\underline{aab} + \underline{aabab}).(\underline{a}+\underline{b})^*$

(b) Two cases: no 01's and one 01

$E_2 = \underline{1}^*\underline{0}^* + \underline{1}^*\underline{0}^*(\underline{01}).\underline{1}^*\underline{0}^*$

3(a)



∴ E is inaccessible

(b) P_0: {C, D, G, H} {A, B, F}

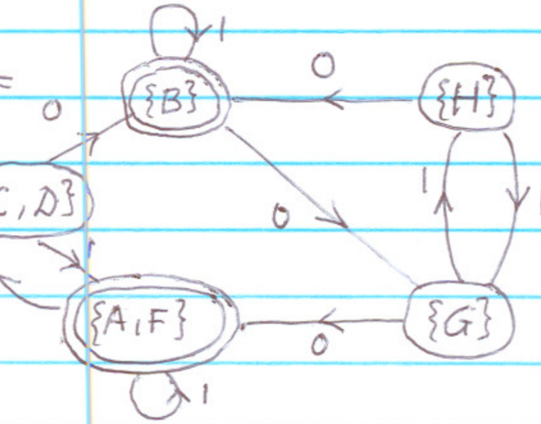
P_1: {C, D} {G, H} {A, B, F}

P_2: {C, D} {G, H} {A, F} {B}

P_3: {C, D} {G} {H} {A, F} {B}

P_4: {C, D} {G} {H} {A, F} {B} = P_3.

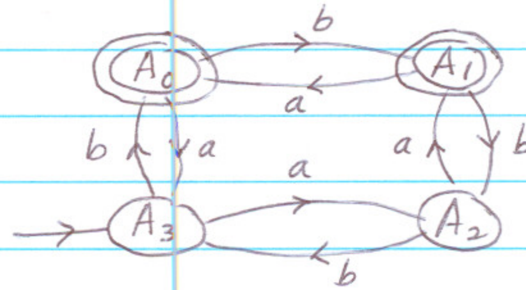
M_R =



4(a) Let $f(\omega) = n_b(\omega) - n_a(\omega) - 1 \pmod 4$ and A_i $(i = 0,1,2,3)$ keep track of $f(\varphi)$ where φ = part of the string processed. Then $f(\lambda) = 0 - 0 - 1 \equiv 3 \pmod 4$. So A_3 will be the accepting state. Also $f(\omega) < 2$ when $f(\omega) = 0$ or $f(\omega) = 1$, so A_0 and A_1 will be the accepting states. Finally

$f(\omega a) = n_b(\omega a) - n_a(\omega a) - 1 = n_b(\omega) - n_a(\omega) - 1 - 1 \equiv f(\omega) - 1 \equiv f(\omega) + 3$

$f(\omega b) = n_b(\omega b) - n_a(\omega b) - 1 = n_b(\omega) + 1 - n_a(\omega) - 1 \equiv f(\omega) + 1$



(b) Let $\omega = babb$

Input: $b \quad a \quad b \quad b$

States: $A_3 \quad A_0 \quad A_3 \quad A_0 \quad A_1$

$\therefore f(\omega) = 1$ according to DFA

Check: $n_b(\omega) - n_a(\omega) - 1 = 3 - 1 - 1 = 1 \pmod 4$ ✓

5 (a) An unreachable production of G is any production which contains a variable that cannot be reached from the starting variable S. A non-terminating production is any production that contains a variable which does not terminate or lead to something that eventually terminates.

(b) $S \to S_1 / S_2$ (This gives the union from S_1 & S_2)

$S_1 \to AS_1b \,/\, AA, \quad A \to a/\lambda$

$S_2 \to bbS_2a \,/\, B, \quad B \to bB \,/\, bbb$

$S_1 \Rightarrow AS_1b \Rightarrow AAS_1bb \Rightarrow \cdots \Rightarrow A^nS_1b^n \Rightarrow A^n.AA.b^n = A^{n+2}b^n$

The A's can be replaced by a or λ to give a^kb^n with $k \le n+2$. So S_1 generates $\{a^kb^n : k < n+3\}$.

$S_2 \Rightarrow bbS_2a \Rightarrow b^4S_2a^2 \Rightarrow \cdots \Rightarrow b^{2n}S_2a^n \Rightarrow b^{2n}Ba^n \Rightarrow b^{2n}.bB.a^n$

$\Rightarrow \cdots \Rightarrow b^{2n}.b^{\ell}.B.a^n \Rightarrow b^{2n+\ell}.bbb.a^n = b^{2n+3+\ell}.a^n = b^k.a^n$

with $k \ge 2n+3$. So S_2 generates $\{b^k.a^n : k > 2n+2\}$

6(a) YES. Let $\varphi \in (B.A^R) \cup (C.A^R)$. Then $\varphi \in B.A^R$ or $\varphi \in C.A^R$. Hence either
$\varphi = \beta.\alpha^R$ with $\beta \in B$ and $\alpha \in A$, or
$\varphi = \gamma.\alpha^R$ with $\gamma \in C$ and $\alpha \in A$.
In the first case $\varphi = \beta.\alpha^R \in (B \cup C).A^R$ because $\beta \in (B \cup C)$ and $\alpha \in A$. And in the second case $\varphi = \gamma.\alpha^R \in$ $\in (B \cup C).A^R$ because $\gamma \in (B \cup C)$ and $\alpha \in A$. So in either case $\varphi \in (B \cup C).A^R$. Hence we will always have $(B.A^R) \cup (C.A^R) \subseteq (B \cup C).A^R$.

(b) NO. Let $A = \{1, 10\}$, $B = \{10\}$ and $C = \{1\}$.
Then $(B \cap C) = \emptyset$, so $(B \cap C).A^R = \emptyset.\{1, 01\} = \emptyset$
Also $B.A^R = \{10\}.\{1, 10\}^R = \{10\}.\{1, 01\} = \{101, 1001\}$
and $C.A^R = \{1\}.\{1, 10\}^R = \{1\}.\{1, 01\} = \{11, 101\}$
$\therefore (B.A^R) \cap (C.A^R) = \{101, 1001\} \cap \{11, 101\} = \{101\}$.
Hence in this particular case
$$(B.A^R) \cap (C.A^R) \nsubseteq (B \cap C).A^R.$$
So it is not always true that for all A, B, C
$$(B.A^R) \cap (C.A^R) \subseteq (B \cap C).A^R$$