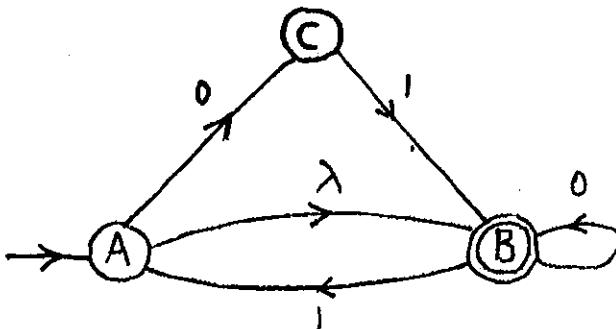


Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit.
BEGIN EACH QUESTION ON A SEPARATE PAGE.

- (15) 1. (a) Define what is the extended transition function of an NFA.

- (b) Convert the NFA on the right into an equivalent DFA.



- (15) 2. Find regular expressions which describe the languages below
 (a) $L_1 = \{\alpha \in \{0,1\}^*: \alpha \text{ contains both } 01 \text{ & } 100 \text{ as substrings}\}$
 (b) $L_2 = \{\beta \in \{0,1\}^*: \beta \text{ has at most two occurrences of } 10\}$

- (20) 3. (a) Find all the inaccessible states in the DFA below.
 (b) Then partition the remaining states into blocks of indistinguishable states and find the reduced machine.

	A	B	C	D	E	F	G	H
0	F	B	F	H	G	A	E	D
1	C	G	B	G	B	C	A	A

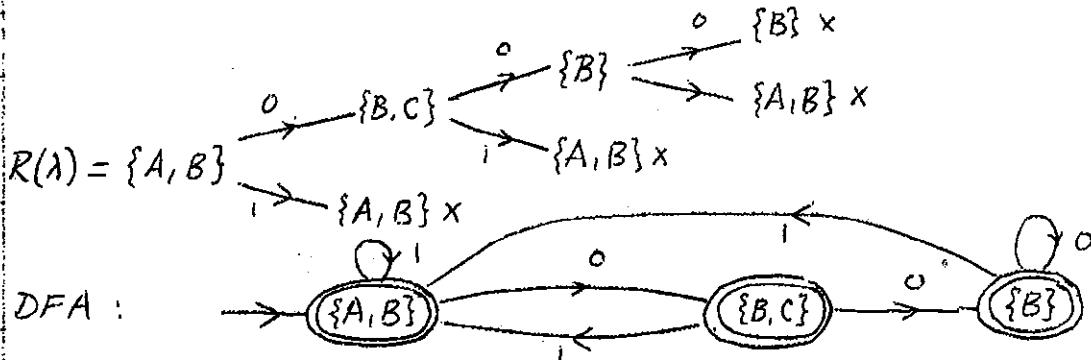
- (15) 4. Find a DFA which accepts precisely the strings in the language
 $L_4 = \{\omega \in \{a,b\}^*: [2n_a(\omega) - 3n_b(\omega) - 1] \pmod 4 < 2\}$
 and then check your DFA with aaba as input.

- (20) 5. (a) Define what are inaccessible productions and non-terminating productions in a CFG G.
 (b) Find a context-free grammar which generates the language
 $L_5 = \{c^k b^n : k > 2n\} \cup \{b^k a^n : k < 2n+3\}.$

- (15) 6. Let A, B and C be languages based on the alphabet {0,1}.
 (a) Is it always true that $(A \cdot C^*) \cup (B \cdot C^*) \subseteq (A \cup B) \cdot C^*$?
 (b) Is it always true that $A \cdot (B - C) \subseteq (A \cdot B) - (A \cdot C)$? Justify your answers completely.

- 1(a) The extended transition function $\Delta^*: Q \times \Sigma^* \rightarrow P(Q)$ of an NFA is defined by $\Delta^*(p, w) = \{q \in Q : w \text{ can lead you from } p \text{ to } q\}$

(b)



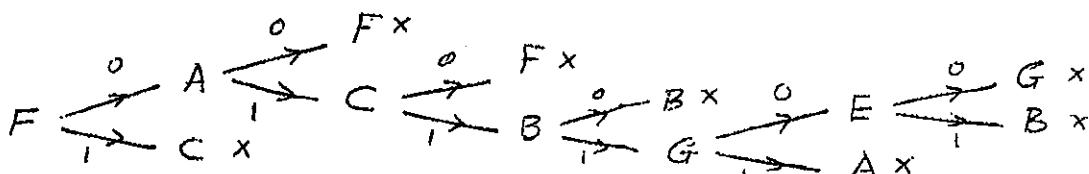
- 2 (a) $\dots 01 \dots 100 \dots, \dots 100 \dots 01 \dots, \dots 0100 \dots, \dots 1001 \dots$

$$E_1 = (0+1)^* (01, (0+1)^* 100 + 100, (0+1)^* 01 + 0100 + 1001)(0+1)^*$$

- (b) No 10's, one 10, two 10's

$$E_2 = 0^{*!} + 0^{*!} \cdot 10 \cdot 0^{*!} + 0^{*!} 10 \cdot 0^{*!} 10 \cdot 0^{*!} = 0^{*!} 0^{*!} 0^{*!}.$$

3 (a)



\therefore D and H are inaccessible states

$$P_0: \{A, B, F\}, \{C, E, G\}$$

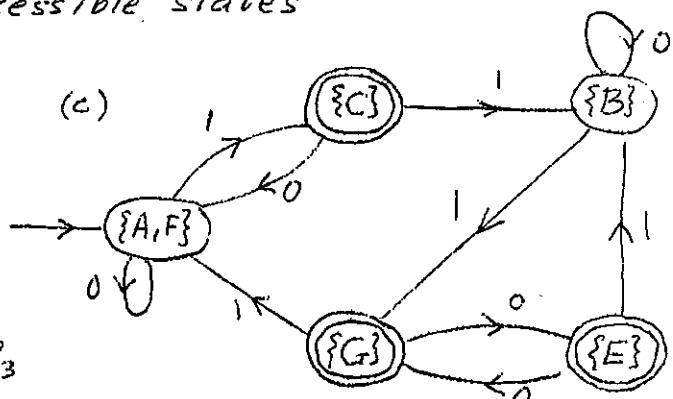
$$P_1: \{A, B, F\}, \{C\}, \{E, G\}$$

$$P_2: \{A, F\}, \{B\}, \{C\}, \{E, G\}$$

$$P_3: \{A, F\}, \{B\}, \{C\}, \{E\}, \{G\}$$

$$P_4: \{A, F\}, \{B\}, \{C\}, \{E\}, \{G\} = P_3$$

(c)

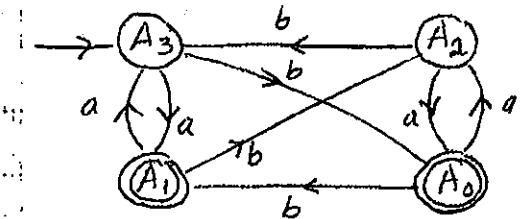


- 4(a) Let $f(w) = 2n_a(w) - 3n_b(w) - 1 \pmod{4}$ and let A_i ($i=0, 1, 2, 3$) be the state that stores the information that $f(w) \equiv i \pmod{4}$.

4(a) Then $f(\lambda) = 2n_a(\lambda) - 3n_b(\lambda) - 1 = 0 - 0 - 1 \equiv 3 \pmod{4}$, So A_3 will be the initial state. A_0 and A_1 will be the accepting states because $0 < 2$ and $1 < 2$. Also

$$f(wa) = 2n_a(wa) - 3n_b(wa) - 1 = 2n_a(w) - 3n_b(w) - 1 + 2 = f(w) + 2 \pmod{4}$$

$$f(wb) = 2n_a(wb) - 3n_b(wb) - 1 = 2n_a(w) - 3n_b(w) - 1 - 3 \equiv f(w) + 1 \pmod{4}$$



(b) input: $w = a \ a \ b \ a$
states: $A_3 \ A_1 \ A_3 \ A_0 \ A_2$

$$\text{check: } 2n_a(w) - 3n_b(w) - 1 = 2(3) - 3 - 1 \equiv 2 \pmod{4}$$

5(a) An inaccessible (or unreachable) production is one which involves a variable that cannot be reached from the starting variable.
A non-terminating production is one that involves a variable that does not terminate or lead to something that eventually terminates.

(b) $S \rightarrow A/B$ gives the union
 $A \rightarrow cCAb/cA/c$ generates $\{c^{2n+1+p}b^n : n \geq 0, p \geq 0\}$
 $B \rightarrow DDBa/DD, D \rightarrow b/\lambda$ generates $\{b^k \cdot a^n : 0 \leq k \leq m+2\}$.
 $S \Rightarrow A \Rightarrow c^2Ab \Rightarrow c^4Ab^2 \Rightarrow \dots \Rightarrow c^{2n}Ab^n \Rightarrow c^{2n}cAb^n \Rightarrow \dots \Rightarrow c^{2n+p}Ab^n \Rightarrow c^{2n+1+p}b^n$,
 $S \Rightarrow B \Rightarrow D^2Ba \Rightarrow D^4Ba^2 \Rightarrow \dots \Rightarrow D^{2n}Ba^n \Rightarrow D^{2n}DDa^n = D^{2n+2}a^n \Rightarrow b^k a^n, 0 \leq k \leq 2n+2$.

6(a) YES. Let $\varphi \in (A \cdot C^*) \cup (B \cdot C^*)$. Then $\varphi \in A \cdot C^*$ or $\varphi \in B \cdot C^*$
So $\varphi = \alpha \cdot \gamma$ for some $\alpha \in A$ and $\gamma \in C^*$,
or $\varphi = \beta \cdot \gamma_2$ for some $\beta \in B$ and $\gamma_2 \in C^*$.
In the first case $\varphi = \alpha \cdot \gamma_1$ with $\alpha \in (A \cup B)$ and $\gamma_1 \in C^*$, so $\varphi \in (A \cup B) \cdot C^*$. And in the second case $\varphi = \beta \cdot \gamma_2$ with $\beta \in (A \cup B)$ and $\gamma_2 \in C^*$, so $\varphi \in (A \cup B) \cdot C^*$. So in either case $\varphi \in (A \cup B) \cdot C^*$. Hence $(A \cdot C^*) \cup (B \cdot C^*) \subseteq (A \cup B) \cdot C^*$

(b) NO. Let $A = \{0, 01\}$, $B = \{1\}$, and $C = \{11\}$. Then $B \cdot C = \{1\}$.
So $A \cdot (B \cdot C) = \{0, 01\} \cdot \{1\} = \{01, 011\}$, $A \cdot B = \{0, 01\} \cdot \{1\} = \{01, 011\}$,
 $A \cdot C = \{0, 01\} \cdot \{11\} = \{011, 0111\}$, and $(A \cdot B) \cdot (A \cdot C) = \{01, 011\} \cdot \{011, 0111\} = \{01\}$
Hence $A \cdot (B \cdot C) = \{01, 011\} \neq \{01\} = (A \cdot B) \cdot (A \cdot C)$. END