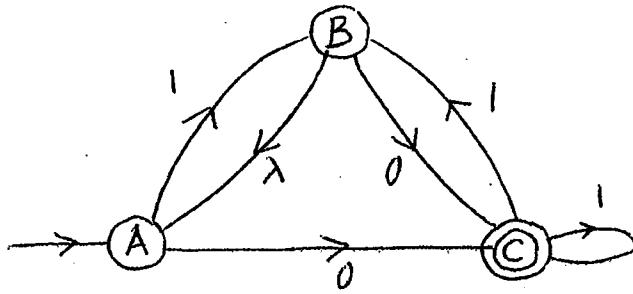


TEST #1 - SPRING 2010TIME: 75 min.

Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

- (15) 1. (a) Define what is a **regular expression** over the alphabet $\{a, b, c\}$.

- (b) Convert the NFA on the right into an equivalent DFA.



- (15) 2. Find regular expressions which describe the languages below.

- (a) $L_1 = \{\alpha \in \{0,1\}^*: \alpha \text{ contains both } 110 \text{ & } 101 \text{ as substrings}\}$
 (b) $L_2 = \{\beta \in \{0,1\}^*: \beta \text{ has exactly two occurrences of } 00\}$

- (20) 3. (a) Find all the **inaccessible states** in the DFA below.
 (b) Then **partition the remaining states** into blocks of indistinguishable states and find the **reduced machine**.

	A	B	$\rightarrow @$	D	E	F	G	H
0	H	G	B	C	B	G	A	B
1	D	B	D	A	G	H	E	A

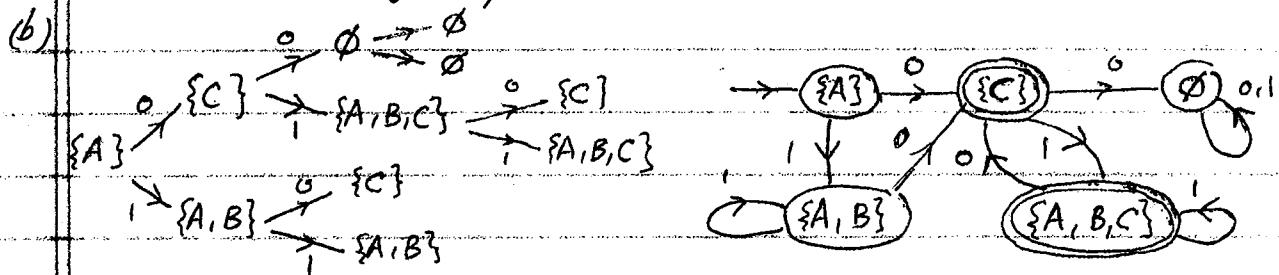
- (15) 4. Find a DFA which accepts precisely the strings in the language $L_4 = \{\omega \in \{a,b\}^*: f(\omega) < 2\}$, where $f(\omega) = [2n_a(\omega) - 3n_b(\omega) - 1] \pmod 4$, and then **check your DFA** with **abbab** as input.

- (20) 5. (a) Define what are **useless productions** of a context-free grammar G.
 (b) Find a context-free grammar which generates the language $L_5 = \{a^k b^{n+1}: k > 2n\} \cup \{a^{k+1} c^n: k < 3n+2\}$.

- (15) 6. Let A, B, and C be languages based on the alphabet $\{0,1\}$.

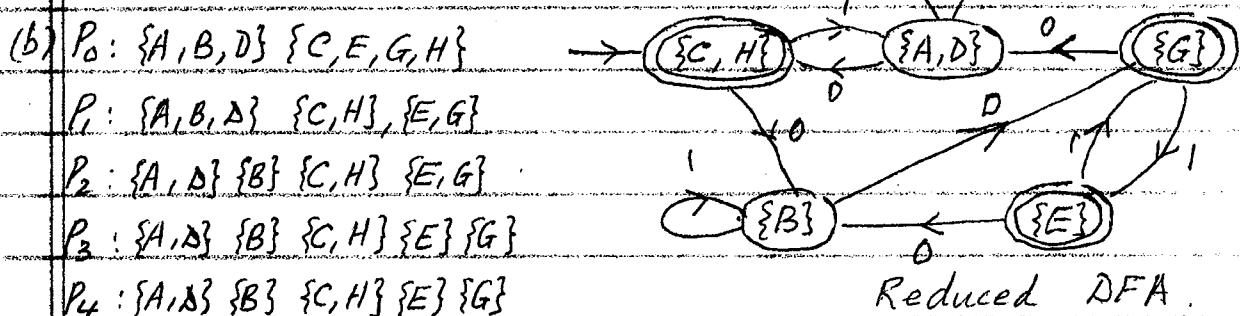
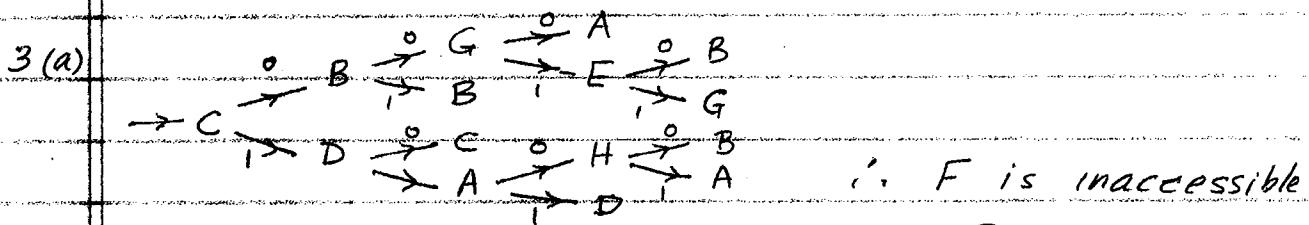
- (a) Is it always true that $(A-B) \cdot C \subseteq (A \cdot C) - (B \cdot C)$?
 (b) Is it always true that $(A \cap B) \cdot C \subseteq (A \cdot C) \cap (B \cdot C)$? Justify your answers completely.

1 (a) A regular expression over $\{a, b, c\}$ is defined recursively as follows (i) a, b, c, λ , and \emptyset are regular expressions. (ii) If E and F are reg. expr. then so are $(E+F)$, (E,F) & (E^*) .



2 (a) $\dots 110 \dots 101 \dots, \dots 101 \dots 110 \dots, \dots 1101 \dots, \dots 10110 \dots$
 $(0+1)^* \cdot (110(0+1)^*101 + 101(0+1)^*110 + 1101 + 10110) \cdot (0+1)^*$

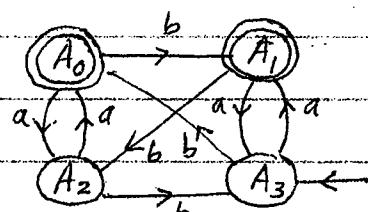
(b) $\dots 00 \dots 00 \dots, \dots 000 \dots$
 $(1+01)^* \cdot 001 \cdot (1+01)^* \cdot 00 \cdot (1+10)^* + (1+01)^* \cdot 000 \cdot (1+10)^*$



4 (a) Let A_i ($i=0, 1, 2, 3$) keep track of the fact that $f(w) \equiv i \pmod{4}$. Since $f(\lambda) = 2n_a(\lambda) - 3n_b(\lambda) - 1 = 0 - 0 - 1 \equiv 3 \pmod{4}$, A_3 will be the initial state. Also, since $0 < 2$ and $1 < 2$, A_0 & A_1 will be the accepting states.

$$4(a) f(wa) = 2n_a(wa) - 3n_b(wa) - 1 = 2n_a(w) - 3n_b(w) - 1 + 2 \\ = f(w) + 2 \pmod{4}$$

$$f(wb) = 2n_a(wb) - 3n_b(wb) - 1 = 2n_a(w) - 3n_b(w) - 1 - 3 \\ = f(w) - 3 \pmod{4} = f(w) + 1 \pmod{4}$$



$$\begin{array}{cccccc} a & b & b & a & b \\ A_3 & A_1 & A_2 & A_3 & A_1 & A_2 \end{array}$$

$$f(abbab) = 2(2) - 3(3) - 1 \\ = -6 \pmod{4} = 2 \pmod{4} \checkmark$$

5(a) A useless production is an unreachable or non-terminating production. An unreachable production is one which involves a variable which cannot be reached from the starting variable. A non-terminating production is one which involves a variable which does not eventually terminate.

$$(b) S \rightarrow A/B$$

$$A \rightarrow CCAb/Cb, C \rightarrow aC/a, \quad \{a^k b^{n+1} : k \geq 2n+1\}$$

$$B \rightarrow DDBc/Da, D \rightarrow a/\lambda. \quad \{a^{k+1} c^n : k \leq 3n+1\}$$

$$S \Rightarrow A \Rightarrow CCAb \Rightarrow \dots \Rightarrow C^{2n}Ab^n \Rightarrow C^{2n}.Cb.b^n \Rightarrow \dots \Rightarrow C^{2n+1}.C.b^{n+1}$$

$$S \Rightarrow B \Rightarrow D^3Bc \Rightarrow \dots \Rightarrow D^{3n}Bc^n \Rightarrow D^{3n}.Da.c^n = D^{3n+1}.a.c^n$$

6(a) No. Let $A = \{\lambda\}$, $B = \{\lambda\}$ and $C = \{\lambda, 1\}$. Then

$$(A-B).C = (\{\lambda\} - \{\lambda\}).\{\lambda, 1\} = \{\lambda\}.\{\lambda, 1\} = \{\lambda, 1\} \text{ and}$$

$$(A.C) - (B.C) = \{\lambda, \{\lambda, 1\}\} - \{\lambda\}.\{\lambda, 1\} = \{\lambda, 1\} - \{\lambda, 1\} = \{\lambda\}.$$

So $(A-B).C \neq (A.C) - (B.C)$ in general.

(b) Yes. Let $\varphi \in (A \cap B).C$. Then $\varphi = \alpha \cdot \gamma$ with $\alpha \in A \cap B$ and $\gamma \in C$. Since $\alpha \in A \cap B$, $\alpha \in A$ and $\alpha \in B$.

So $\varphi = \alpha \cdot \gamma$ with $\alpha \in A$ & $\gamma \in C$ and $\varphi = \alpha \cdot \gamma$ with $\alpha \in B$ & $\gamma \in C$. Hence $\varphi \in A.C$ and $\varphi \in B.C$.

Thus $\varphi \in (A.C) \cap (B.C)$. Hence $(A \cap B).C \subseteq (A.C) \cap (B.C)$.