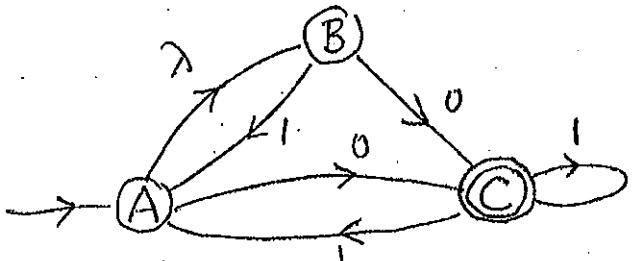


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1. (a) Define the *extended transition function* δ^* of an DFA M .

- (b) Convert the NFA on the right into an *equivalent DFA*.



- (15) 2. Find *regular expressions* which describe the languages below.

(a) $L_1 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains both } ba \text{ and } aab \text{ as substrings}\}$

(b) $L_2 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains at most one occurrence of the string } 00\}$

- (20) 3. (a) Define what it means for two states p & q to be *indistinguishable* in a DFA M .
(b) Partition the states of the DFA below into *blocks of indistinguishable states* & then find the *reduced machine*.

	A	B	$\rightarrow C$	D	E	F	G
0	C	B	A	C	G	A	E
1	F	G	D	B	B	B	A

- (15) 4. (a) Let $f(\varphi) = [2n_b(\varphi) - 3 \cdot \{n_a(\varphi) + 3\}] \pmod{4}$. Find a DFA which accepts the language $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is } 1 \text{ or } 3\}$.
(b) If $\varphi = babab$ find $f(\varphi)$ and then check your DFA with $babab$ as input.

- (20) 5. (a) Find a *context-free grammar* which generates the language $L_5 = \{a^k b^n : k \geq 2n+1\} \cup \{b^k c^n : k \leq 2n+3\}$.
(b) Using your CFG, find *derivations* for each of the strings $a^6 b^2$ and $b^5 c^2$.

- (15) 6. Let A , B , and C be languages based on the *alphabet* $\{0,1\}$.
(a) Is it always true that $(A \cdot C) \cup (B \cdot C) \subseteq (A \cup B) \cdot C$?
(b) Is it always true that $(A \cdot C) \cap (B \cdot C) \subseteq (A \cap B) \cdot C$?
Justify your answers completely.

Solutions to Test #1

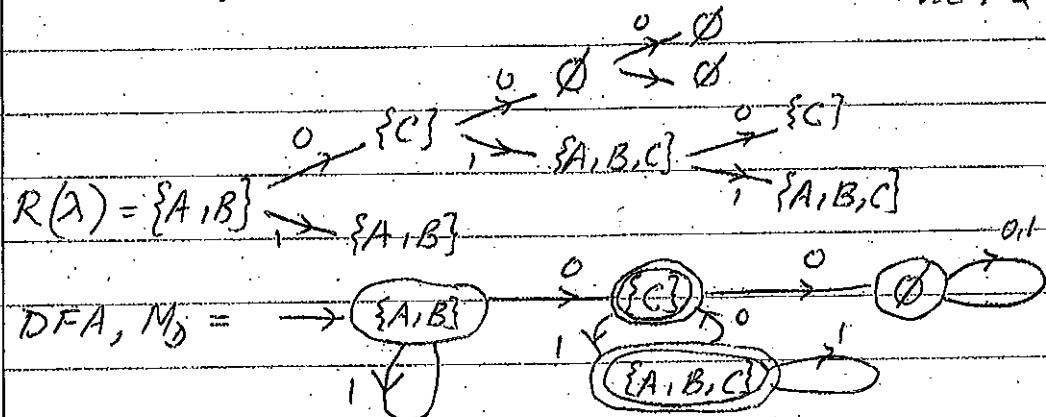
Spring 2014

1(a) The extended transition function $\delta^*: Q \times T^* \rightarrow Q$, of a DFA

$M = \langle Q, T, \delta, q_0, A \rangle$ is defined recursively as follows.

(i) $\delta^*(q, \lambda) = q$ & (ii) $\delta^*(q, \varphi a) = \delta(\delta^*(q, a), a)$ for each $q \in Q$, $a \in T$ & $\varphi \in T^*$.

(b)



2(a) ...ba...aab..., ...aab...ba..., ...baab..., "aaba..."
 $E_1 = (a+b)^* (ba(a+b)^* aab + aab(a+b)^* ba + baab + aaba)(a+b)^*$

(b) ...no 00's ..., ...one 00 ...

$$E_2 = (1+01)^*, (1+0)^* + (1+01)^* 00, (1+10)^*$$

Another answer: $E_2' = (1+01)^* (1+0+00) \cdot (1+10)^*$

3(a) Two states p & q are indistinguishable in a DFA M if
 for each $\varphi \in T^*$, $\delta^*(p, \varphi) \in A(M) \Leftrightarrow \delta^*(q, \varphi) \in A(M)$.

$$P_0 : \{A, B, C\} \quad \{D, E, F, G\}$$

$$P_1 : \{A, B, C\} \quad \{D, F\} \quad \{E, G\}$$

$$P_2 : \{A, C\} \quad \{B\} \quad \{D, F\} \quad \{E, G\}$$

$$P_3 : \{A, C\} \quad \{B\} \quad \{D, F\} \quad \{E\} \quad \{G\}$$

$$P_4 : \{A, C\} \quad \{B\} \quad \{D, F\} \quad \{E\} \quad \{G\} = P_3.$$

q_0	$\rightarrow \{A, C\}$	$\{B\}$	$\{D, F\}$	$\{E\}$	$\{G\}$
0	$\{A, C\}$	$\{B\}$	$\{A, C\}$	$\{G\}$	$\{E\}$
1	$\{D, F\}$	$\{G\}$	$\{B\}$	$\{B\}$	$\{A, C\}$

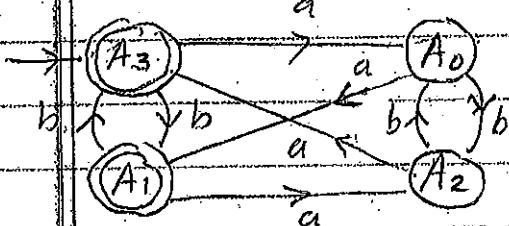
4(a) Let A_i ($i=0, 1, 2, 3$) keep track of the fact that $f(\varphi) \equiv i \pmod{4}$

$$f(\lambda) = 2n_b(\lambda) - 3[n_a(\lambda) + 3] = 0 - 3(3) = -9 \equiv -1 \equiv 3 \pmod{4}$$

So A_3 will be the initial state. Also A_1 & A_3 will be the accepting states because $\varphi \in L_4$ iff $f(\varphi) = 1$ or $3 \pmod{4}$

$$\begin{aligned} f(\varphi a) &= 2n_b(\varphi a) - 3[n_a(\varphi a) + 3] = 2n_b(\varphi) - 3[n_a(\varphi) + 3] - 3 \\ &\equiv f(\varphi) - 3 \equiv f(\varphi) + 1 \pmod{4} \end{aligned}$$

$$\begin{aligned} f(\varphi b) &= 2n_b(\varphi a) - 3[n_a(\varphi b) + 3] = 2n_b(\varphi) - 3[n_a(\varphi) + 3] + 2 \\ &\equiv f(\varphi) + 2 \pmod{4} \end{aligned}$$



$$(b) f(babab) = 2(3) - 3(2+3) \equiv 3 \pmod{4}$$

input: $b \ a \ b \ a \ b \ \sqcup$
states: $A_3, A_1, A_2, A_0, A_1, A_3 \checkmark$

5(a) $S \rightarrow E/F, E \rightarrow aaEb/A, A \rightarrow aA/a, F \rightarrow BBFC_c/BBB, B \rightarrow b/\lambda$

(b) (i) $S \rightarrow E \rightarrow aaEb \rightarrow aaaaEb \rightarrow aaaaAabb$

$$\Rightarrow aaaa aAabb \Rightarrow aaaaaabb = a^6b^2$$

(ii) $S \rightarrow F \Rightarrow BBFC_c \Rightarrow BBBBFcc \Rightarrow BBBB BBBcc \Rightarrow \lambda.BBBBBBcc$

$$\Rightarrow \lambda.\lambda.BBBBBBcc \Rightarrow bBBBBBcc \Rightarrow bbb BBBcc \Rightarrow bbbb Bcc \Rightarrow bbbbb Bcc \Rightarrow b^5c^2.$$

6(a) YES. Let $\varphi \in (A.C) \cup (B.C)$. Then $\varphi \in A.C$ or $\varphi \in B.C$.

Now if $\varphi \in A.C$, then $\varphi = \alpha.\gamma$ with $\alpha \in A$ & $\gamma \in C$. So

$\varphi = \alpha.\gamma \in (A \cup B).C$ because $\alpha \in (A \cup B)$ & $\gamma \in C$.

And if $\varphi \in B.C$, then $\varphi = \beta.S$ with $\beta \in B$ & $S \in C$. So

$\varphi = \beta.S \in (A \cup B).C$ because $\beta \in (A \cup B)$ & $S \in C$. Hence

$\varphi \in (A \cup B).C$. $\therefore (A.C) \cup (B.C) \subseteq (A \cup B).C$

(b) Let $A = \{0\}$, $B = \{01\}$, and $C = \{\lambda, 1\}$. Then

$$(A \cup B) \cap (B.C) = \{0\} \cup \{\lambda, 1\} \cap \{01\}, \{\lambda, 1\} = \{0, 01\} \cap \{01, 011\} = \{01\}$$

and $(A \cap B).C = (\{0\} \cap \{01\}).\{\lambda, 1\} = \emptyset \cdot \{\lambda, 1\} = \emptyset$. Since

$\{01\} \not\subseteq \emptyset$, $(A \cup B) \cap (B.C) \not\subseteq (A \cap B).C$. Hence $(A.C) \cup (B.C)$ is not always a subset of $(A \cup B).C$.