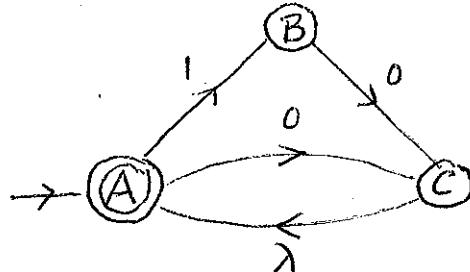


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1.(a) Define what is a *regular expression* over the alphabet $V = \{0,1,2\}$.

- (b) Convert the *NFA* M on the right into an equivalent *DFA*, M_D .



- (15) 2. Find *regular expressions* which describe the languages below.

- (a) $L_1 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains both } aab \text{ and } aba \text{ as substrings}\}$
(b) $L_2 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains exactly 2 occurrences of the string } 00\}$

- (20) 3. (a) Define what it means for a state p to be *inaccessible* in a *DFA*, M.

- (b) Partition the states of the *DFA*, M below into *blocks of indistinguishable states* and then find the equivalent *reduced machine*, M_R .

	A	B	C	D	E	F	G
0	F	B	F	C	G	C	E
1	B	G	D	B	B	A	F

- (15) 4. (a) Let $f(\varphi) = \{2 \cdot n_a(\varphi) - n_b(\varphi) - 3\} \pmod{4}$. Find a *DFA*, M which accepts the language, $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is even}\}$.

- (b) If $\varphi = baabb$ find $f(\varphi)$ and then check your *DFA* with $baabb$ as input .

- (20) 5. (a) Find a *context-free grammar* G which generates the language

$$L_5 = \{a^k b^n : k \geq 3n + 2\} \cup \{b^k c^n : k \leq 2n + 1\}.$$

- (b) Find *derivations* in G for each of the strings: (i) $a^6 b^1$ and (ii) $b^4 c^2$.

- (15) 6. Let A, B, and C be languages based on the alphabet $\{0,1\}$.

- (b) Is it always true that $(A \cap B)^* \cdot C \subseteq (A^* \cdot C) \cap (B^* \cdot C)$?

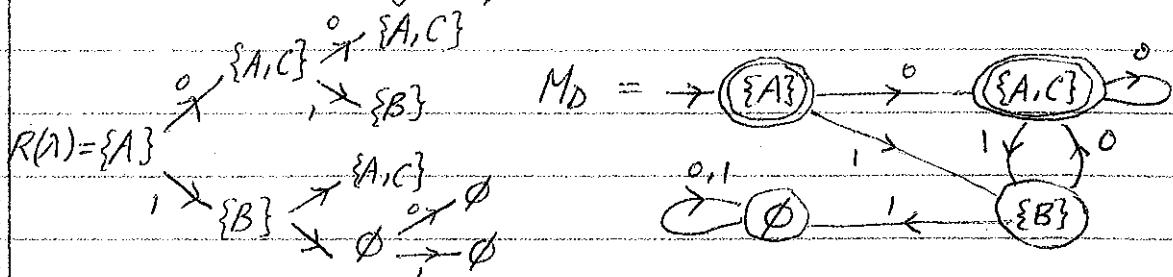
- (a) Is it always true that $(A - B) \cdot C \subseteq (A \cdot C) - (B \cdot C)$?

Justify your answers completely.

MAD 3512 - Theory of Algorithms Florida Int'l Univ.
 Solutions to Test #1 Spring 2015

- (a) A regular expression over $\{0, 1, 2\}$ is defined recursively as follows. (i) $0, 1, 2, \lambda$ and \emptyset are regular expressions
 (ii) if $E & F$ are reg. expr., then so are $(E+F), (E.F), \& (E^*)$

(b)



2 (a) ...aab...aba..., ...aba...aab..., ...aaba..., abaab...

$$(a+b)^*(aab, (a+b)^*.aba + aba.(a+b)^*.aab + aaba + abaab). (a+b)^*$$

(b) ...001...00..., ...000...

$$(1+01)^*.001.(1+01)^*.00.(1+10)^* + (1+01)^*.000.(1+10)^*$$

3 (a) The state p in a DFA M is inaccessible if there is no string $q \in T^*$ such that $\delta^*(q_0, q) = p$. [q_0 = init. state of M]

(b) $P_0 : \{B, C, F\} \quad \{A, D, E, G\} = \text{initial partition}$

$P_1 : \{B, C, F\} \quad \{A, D\} \quad \{E, G\}$

$P_2 : \{C, F\} \quad \{B\} \quad \{A, D\}$

$P_3 : \{C, F\} \quad \{B\} \quad \{A, D\} \quad \{E\} \quad \{G\}$

$P_4 : \{C, F\} \quad \{B\} \quad \{A, D\} \quad \{E\} \quad \{G\} = P_3 = \text{final partition}$

$$T \xrightarrow{Q^D} \{C, F\} \quad \{B\} \quad \{A, D\} \quad \{E\} \quad \{G\}$$

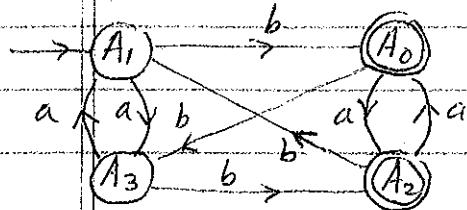
0	$\{C, F\}$	$\{B\}$	$\{C, F\}$	$\{G\}$	$\{G\}$
1	$\{A, D\}$	$\{G\}$	$\{B\}$	$\{B\}$	$\{C, F\}$

4 (a) Let A_i ($i=0, 1, 2, 3$) keep track of the fact that $f(q) \equiv i \pmod{4}$.

Then $f(1) = 2n_a(\lambda) - n_b(\lambda) - 3 = 2(0) - 0 - 3 \equiv 1 \pmod{4}$. So A_1 will be the initial state of M . A_2 & A_4 will be the accepting states of M because $f(q)$ is even when $f(q) \equiv 0$ or 2 .

$$4(a) \quad f(\varphi a) = 2n_a(\varphi a) - n_b(\varphi a) - 3 = 2[n_a(\varphi) + 1] - n_b(\varphi) - 3 \\ = [2n_a(\varphi) - n_b(\varphi) - 3] + 2 \equiv f(\varphi) + 2 \pmod{4}$$

$$f(\varphi b) = 2n_a(\varphi b) - n_b(\varphi b) - 3 = 2n_a(\varphi) - [n_b(\varphi) + 1] - 3 \\ = [2n_a(\varphi) - n_b(\varphi) - 3] - 1 \equiv f(\varphi) + 3 \pmod{4}.$$



$$4(b) \quad f(baabb) = 2(2) - 3 - 3 \\ = -2 \equiv 2 \pmod{4}$$

Check: $\begin{matrix} & b & a & a & b & b & \end{matrix}$

$A_1 A_0 A_2 A_0 A_3 A_2 \checkmark$

$$5(a) \quad S \rightarrow P/Q, \quad P \rightarrow aaaPb/A, \quad A \rightarrow aA/aa, \quad Q \rightarrow BBQc/B, \quad B \rightarrow b/\lambda$$

$$(b)(i) \quad S \Rightarrow P \Rightarrow aaaPb \Rightarrow aaaAb \Rightarrow aaaaAb \Rightarrow aaaaaaab$$

$$(ii) \quad S \Rightarrow Q \Rightarrow BBQc \Rightarrow BBBBQcc \Rightarrow BBBBBBcc \Rightarrow \lambda BBBBBBcc$$

$$\Rightarrow \lambda bBBBcc \Rightarrow \lambda bbBBcc \Rightarrow \lambda bbbBcc \Rightarrow bbbbcc.$$

Note: $S \Rightarrow P \Rightarrow a^3Pb \Rightarrow a^6Pb^2 \dots \Rightarrow a^{3n}Pb^n \Rightarrow a^{3n}Ab^n \Rightarrow \dots$

$$\Rightarrow a^{3n}a^\ell Ab^n \Rightarrow a^{3n+\ell}aa.b^n = a^{3n+2+\ell}b^n, \quad (n, \ell \geq 0)$$

$$S \Rightarrow Q \Rightarrow B^2Qc \Rightarrow B^4Qc^2 \Rightarrow \dots \Rightarrow B^{2n}Qc^n \Rightarrow B^{2n}Bc^n = B^{2n+1}c^n$$

$$\Rightarrow b^k c^n \text{ with } k \leq 2n+1 \text{ (replace } k \text{ B's by b's & rest by } \lambda's\text{.)}$$

6(a) YES. Let $\varphi \in (A \cap B)^*.C$. Then $\varphi = \alpha_1 \alpha_2 \dots \alpha_k \gamma$ where $\alpha_i \in A \cap B$ and $\gamma \in C$. Since $\alpha_i \in A \cap B$, $\alpha_i \in A$ and $\alpha_i \in B$ for each $i = 1, \dots, k$. So $\varphi = \alpha_1 \dots \alpha_k \gamma \in A^*.C$ and $\varphi = \alpha_1 \dots \alpha_k \gamma \in B^*.C$. Hence $\varphi \in (A^*.C) \cap (B^*.C)$.

Thus $(A \cap B)^*.C \subseteq (A^*.C) \cap (B^*.C)$.

(b) NO. Let $A = \{1\}$, $B = \{10\}$, and $C = \{1, 01\}$. Then

$$(A - B).C = (\{1\} - \{10\}) \cdot \{1, 01\} = \{1\} \cdot \{1, 01\} = \{11, 101\}$$

$$\text{Also } A.C = \{1\} \cdot \{1, 01\} = \{11, 101\} \text{ and}$$

$$B.C = \{10\} \cdot \{1, 01\} = \{101, 1001\}$$

$$\text{Thus } A.C - B.C = \{11, 101\} - \{101, 1001\} = \{11\}. \text{ Since}$$

$$(A - B).C = \{11, 101\} \not\subseteq \{11\} = A.C - B.C, \text{ it follows that}$$

it is not always true that $(A - B).C \subseteq A.C - B.C$. END