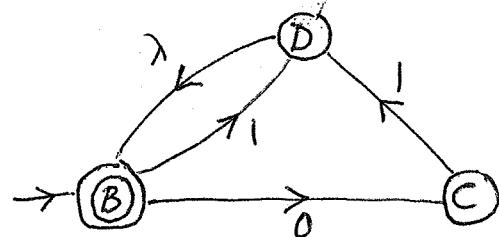


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1.(a) Define what it means for state q in a DFA $M = \langle Q, T, \delta, q_0, A \rangle$ to be *inaccessible*.
 (b) Let M be the NFA on the right. Find an NFA M' which *recognizes* $L(M)^c$.



- (15) 2. Find *regular expressions* which describe the languages below.
 (a) $L_1 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains both } 110 \text{ and } 010 \text{ as substrings}\}$
 (b) $L_2 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains at most one occurrence of the string } bb\}$.

- (20) 3. (a) Define what it means for 2 states p & q to be *indistinguishable* in a DFA M .
 (b) Partition the states of the DFA, M below into *blocks of indistinguishable states* and then find the equivalent *reduced machine*, M_R .

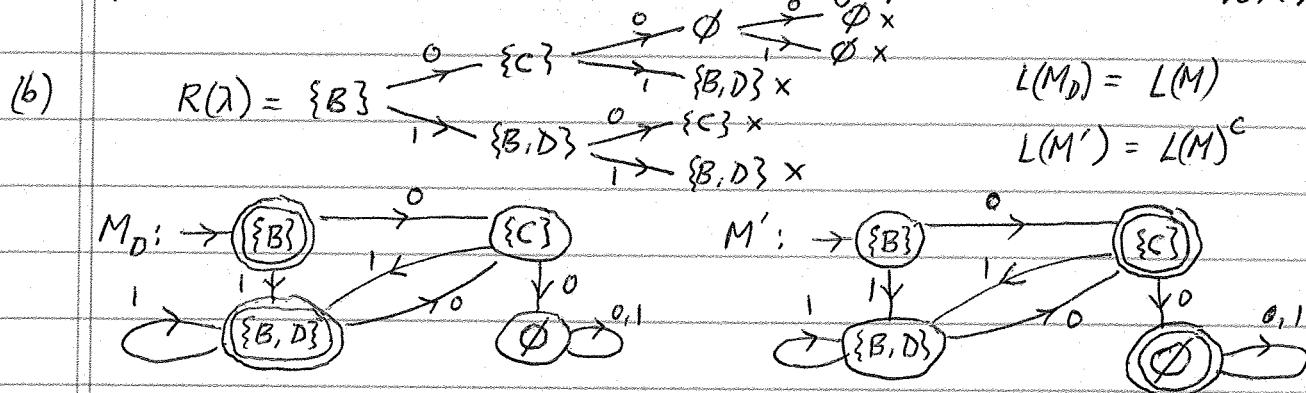
	A	B	C	D	E	F	G
0	B	G	F	B	B	E	D
1	D	B	D	C	G	C	E

- (15) 4. (a) Let $f(\varphi) = [2.n_b(\varphi) + 3.n_a(\varphi) - 1] \pmod{4}$. Find a DFA, M which accepts the language, $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is odd}\}$.
 (b) If $\varphi = baa$ find $f(\varphi)$ and then check your DFA with bba as input .

- (20) 5. (a) Find a *context-free grammar* G which generates the language $L_5 = \{a^k b^n : k \geq 2n + 3\} \cup \{b^k c^n : k \leq 3n + 1\}$.
 (b) Find *derivations* in G for each of the strings: (i) $a^6 b^1$ and (ii) $b^3 c^1$.

- (15) 6. Let A , B , and C be languages based on the *alphabet* $\{0,1\}$.
 (a) Is it always true that $(A^*. B) \cup (A^*. C) \subseteq A^*. (B \cup C)$?
 (b) Is it always true that $(A.B) \cap (A.C) \subseteq A.(B \cap C)$? (Justify your answers.)

1(a) q is inaccessible if there is no string $\varphi \in T^*$ such that $\delta(q_0, \varphi) = q$.



2(a) $\dots 110 \dots 010 \dots, \dots 010 \dots 110 \dots, \dots 11010 \dots \dots = (0+1)^*$

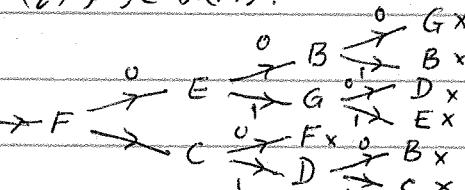
$$E_1 = (0+1)^*(110, (0+1)^*, 010 + 010, (0+1)^*, 110 + 11010), (0+1)^*$$

(b) $E_2 = (a+ba)^* + (a+ba)^*, b + (a+ba)^*, bb \cdot (a+ab)^*$
 If non-empty, begins with "a" or "b" and ends in "a"; — ends in "a" "b".
 If the prefix of "bb" is non-empty, we must end it with an "a";
 If the suffix of "bb" is non-empty, we must begin it with an "a".

3(a) p & q are indistinguishable in M if for each $\varphi \in T^*$ we have
 $\delta^*(p, \varphi) \in A(M) \Leftrightarrow \delta^*(q, \varphi) \in A(M)$.

(b) First check M for any
 inaccessible states.

So A is inaccessible.



$P_0 : \{B, C, D\} \quad \{E, F, G\}$ (initial partition into $Q-A$ & A)

$P_1 : \{B, C\} \quad \{D\} \quad \{E, G\} \quad \{F\}$ b.c. $B \xrightarrow{0} G \& D \xrightarrow{0} B$; $E \xrightarrow{0} B \& F \xrightarrow{0} E$

$P_2 : \{B\} \{C\} \{D\} \{E\} \{F\} \{G\}$ b.c. $B \xrightarrow{0} G \& C \xrightarrow{0} F$ and $E \xrightarrow{0} B \& G \xrightarrow{0} D$

$P_3 : \{B\} \{C\} \{D\} \{E\} \{F\} \{G\} = P_2$ (can also say P_2 can't get any finer.)

$M_R :$	$\{B\}$	$\{C\}$	$\{D\}$	$\{E\}$	$\{F\}$	$\{G\}$
0	$\{G\}$	$\{F\}$	$\{B\}$	$\{B\}$	$\{E\}$	$\{D\}$
1	$\{B\}$	$\{D\}$	$\{C\}$	$\{G\}$	$\{C\}$	$\{E\}$

4(a) Let A_i ($i=0,1,2,3$) keep track of the fact that $f(\varphi) = i \pmod{4}$. Then
 $f(\lambda) = [2n_b(\lambda) + 3n_a(\lambda) - 1] \pmod{4} = (0+0-1) \pmod{4} = 3 \pmod{4}$.

4(a) So A_3 will be the initial state & A_1 and A_3 will be accepting states because 1 & 3 are odd. Let's see what an extra a or b will do.

$$f(\varphi a) = [2n_b(\varphi a) + 3n_a(\varphi a) - 1] \pmod{4}$$

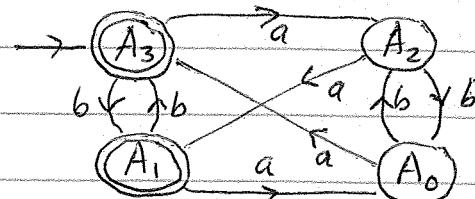
$$= [2n_b(\varphi) + 3n_a(\varphi) - 1] + 3 = f(\varphi) + 3 \pmod{4}.$$

$$f(\varphi b) = [2n_b(\varphi b) + 3n_a(\varphi b) - 1] \pmod{4}$$

$$= [2n_b(\varphi) + 3n_a(\varphi) - 1] + 2 = f(\varphi) + 2 \pmod{4}.$$

$$4.(b) f(baa) = 2n_b(baa) + 3n_a(baa) - 1 \pmod{4}$$

$$= 2(1) + 3(2) - 1 = 7 = 3 \pmod{4}.$$



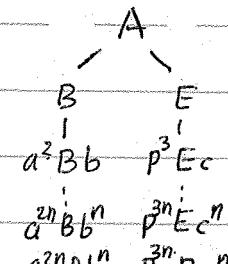
DFA for L_4

<u>b</u>	<u>a</u>	<u>a</u>	<u>...</u>
$\rightarrow A_3$	A_1	A_0	$A_3 \checkmark$

5(a) $\rightarrow A$, $A \rightarrow B|E$, (gives union)

$B \rightarrow aaBb|D$, $D \rightarrow aD|aaa$, (gives $\{a^k b^n : k \geq 2n+3\}$)

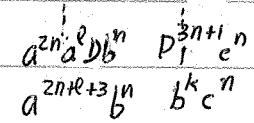
$E \rightarrow PPPE_c|P$, $P \rightarrow b|\lambda$. (gives $\{b^k c^n : k \leq 3n+1\}$)



(b) $\rightarrow A \Rightarrow B \Rightarrow aaBb \Rightarrow aaD b \Rightarrow aaaD b \Rightarrow aaaa aab = a^6 b^1$.

$\rightarrow A \Rightarrow E \Rightarrow P \Rightarrow PPPE_c \Rightarrow PPPP_c \Rightarrow \lambda PPP_c \Rightarrow \lambda b PPP_c$

$\Rightarrow \lambda b b P_c \Rightarrow \lambda b b b c = b^3 c^1$.



6(a) YES. Let $\varphi \in (A^* \cdot B) \cup (A^* \cdot C)$. Then $\varphi \in A^* \cdot B$ or $\varphi \in A^* \cdot C$.

So $\varphi = \alpha \cdot \beta$ where $\alpha \in A^*$ and $\beta \in B$ or $\varphi = \alpha' \cdot \gamma$ where $\alpha' \in A$ and $\gamma \in C$. Now in the first case, we have

$\varphi = \alpha \cdot \beta \in A^*(B \cup C)$ because $\alpha \in A$ and $\beta \in B \subseteq B \cup C$.

And in the case, we have

$\varphi = \alpha' \cdot \gamma \in A^*(B \cup C)$ because $\alpha' \in A$ and $\gamma \in C \subseteq B \cup C$.

So $\varphi \in A^*(B \cup C)$ in either case. $\therefore (A^* \cdot B) \cup (A^* \cdot C) \subseteq A^*(B \cup C)$.

(b) NO. Let $A = \{0, 01\}$, $B = \{1\}$, and $C = \{11\}$. Then

$$A \cdot B = \{0, 01\} \cdot \{1\} = \{01, 011\} \text{ and } A \cdot C = \{0, 01\} \cdot \{11\} = \{011, 0111\}$$

$$\text{So } (A \cdot B) \cap (A \cdot C) = \{01, 011\} \cap \{011, 0111\} = \{011\}.$$

$$\text{Now } A \cdot (B \cap C) = A \cdot (\{1\} \cap \{11\}) = \{0, 01\}, \emptyset = \emptyset$$

$$\text{Since } (A \cdot B) \cap (A \cdot C) = \{011\} \neq \emptyset = A \cdot (B \cap C), \text{ it}$$

follows that it is not always true that $(A \cdot B) \cap (A \cdot C) \subseteq A \cdot (B \cap C)$.

$B = \boxed{1}$

$C = \boxed{11}$