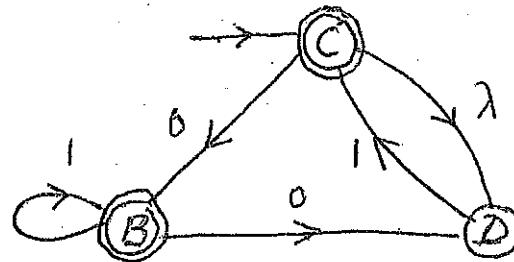


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1. (a) Define what is the *language recognized* by a DFA, $M = \langle Q, T, \delta, q_0, A \rangle$.
 (b) Let M be the NFA on the right. Find a DFA M_D which is *equivalent* to M .



- (15) 2. Find *regular expressions* which describe the languages below.
 (a) $L_1 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains both } bba \text{ and } ab \text{ as substrings}\}$
 (b) $L_2 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains at most one occurrence of the string } 01\}$.

- (20) 3. (a) Define what it means for a state q to be *inaccessible* in a DFA M .
 (b) Partition the states of the DFA, M below into *blocks of indistinguishable states* and then find the equivalent *reduced machine*, M_R .

	(A)	B	$\rightarrow C$	(D)	(E)	F	(G)
0	F	A	D	F	F	G	F
1	B	C	B	C	G	F	E

- (15) 4. (a) Let $f(v) = [2.n_b(v) - 3.n_a(v) - 1] \pmod{4}$. Find a DFA, M which accepts the language, $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) = 2\}$.
 (b) If $\varphi = aaba$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

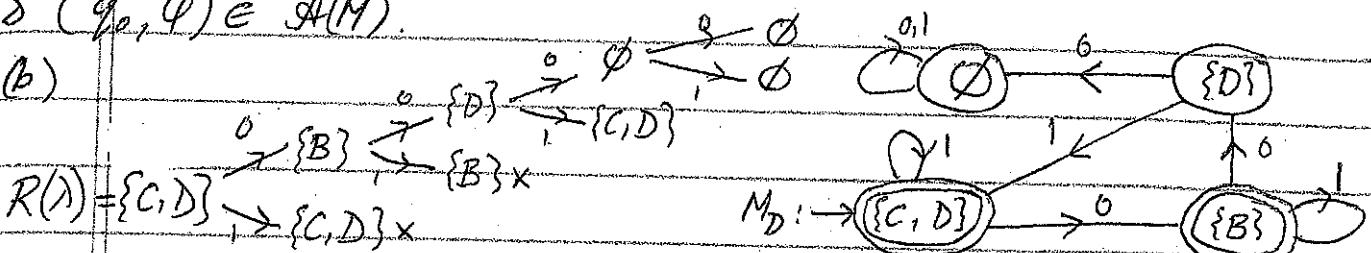
- (20) 5. (a) Find a *context-free grammar* G which generates the language $L_5 = \{a^k b^n: n \geq 2k + 3\} \cup \{b^k c^n: n \leq 3k+2\}$.
 (b) Find *derivations* in G for each of the strings: (i) $a^1 b^6$ and (ii) $b^1 c^3$.

- (15) 6. Let A , B , and C be languages based on the *alphabet* $\{0,1\}$.
 (a) Is it always true that $(A \cap B)C^* \subseteq (A.C^*) \cap (B.C^*)$?
 (b) Is it always true that $(A.C) - (B.C) \subseteq (A-B).C$? (Justify your answers.)

1(a) The language recognized by the DFA M is defined by

$$L(M) = \{\varphi \in T^*: \delta^*(q_0, \varphi) \in A(M)\}, \text{ i.e. } \varphi \in L(M) \iff \delta^*(q_0, \varphi) \in A(M).$$

(b)



$R(\lambda) = \{C, D\}^*$

2.(a) ... bba...ab..., ...ab...bba..., ...babab..., or ... abba...

Reg. expr. for L_1 is $(a+b)^*(bba(a+b)^*ab + ab(a+b)^*bb + bbab + abba)$.

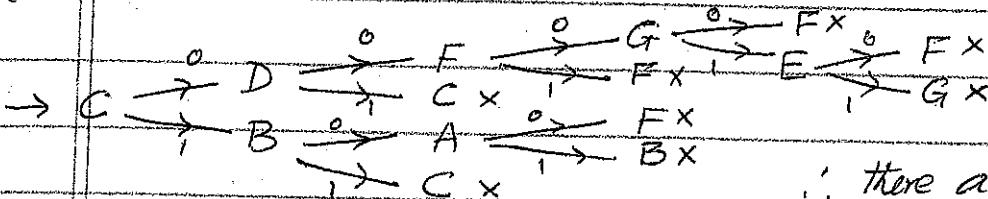
(b) 11...100...0 no 01's or 11...100...0.01.1...110...0 $(a+b)^*$.

Reg. expr. for L_2 is $1^*0^* + 1^*0^*, 01, 1^*0^*$

3(a) A state q in a DFA M is inaccessible if there is no string

$\varphi \in T^*$ such that $\delta^*(q_0, \varphi) = q$.

(b) Let us check first for inaccessible states



: there are no inacc. states.

$P_0: \{B, C, F\} \quad \{A, D, E, G\}$

$P_1: \{B, C, F\} \setminus \{A, D\}, \{E, G\} \quad M_R: \rightarrow \{B, C\} \quad \{F\} \quad (\{A, D\}) \quad (\{E, G\})$

$P_2: \{B, C\}, \{F\} \setminus \{A, D\}, \{E, G\}$

$P_3: \{B, C\}, \{F\}, \{A, D\}, \{E, G\} = P_2$

$0 \mid \{A, D\} \quad \{E, G\} \quad \{F\} \quad \{F\}$

$1 \mid \{B, C\} \quad \{F\} \quad \{B, C\} \quad \{E, G\}$

4(a) Let A_i ($i=0,1,2,3$) keep track of the fact that $f(\varphi) \equiv i \pmod{4}$.

$\text{Then } f(\lambda) = 2n_b(\lambda) - 3n_a(\lambda) - 1 = 0 - 0 - 1 = -1 \equiv 3 \pmod{4}$

So A_3 will be the initial state of our DFA. Also A_2 will be the only accepting state because φ is in $L \iff f(\varphi) \equiv 2 \pmod{4}$.

4(a) ... Now $f(\varphi a) = [2n_b(\varphi a) - 3n_a(\varphi a) - 1] \pmod{4}$ $\rightarrow A_3 \xrightarrow{a} A_0$
 $= [2n_b(\varphi) - 3n_a(\varphi) - 1] - 3 \pmod{4} \equiv f(\varphi) + 1 \pmod{4}$ $b \downarrow b \quad b \downarrow b$
And $f(\varphi b) = [2n_b(\varphi b) - 3n_a(\varphi b) - 1] \pmod{4}$
 $= [2n_b(\varphi) - 3n_a(\varphi) - 1] + 2 \pmod{4} = f(\varphi) + 2 \pmod{4}$ DFA for L₄

4(b) $f(aaba) = 2(1) - 3(3) - 1$ In our DFA: $a \quad a \quad b \quad a \quad \Sigma$
 $= 2 - 9 - 1 = -8 \equiv 0 \pmod{4}$ $\rightarrow A_3 \quad A_0 \quad A_1 \quad A_3 \quad A_0$

5(a) $\rightarrow S, S \rightarrow A \mid B$ (this gives the union)

$$A \rightarrow aAb \mid E, E \rightarrow Eb \mid bbb \text{ (gives } \{a^k b^n : n \geq 2k+3\})$$

$$B \rightarrow bBCCC \mid CC, C \rightarrow c \mid \lambda \text{ (gives } \{b^k c^n : n \leq 3k+2\})$$

$$(b) \rightarrow S \rightarrow A \rightarrow aAb \rightarrow aEbb \rightarrow aEbbb \rightarrow \overbrace{abbbb}^{a'b^6} \overbrace{bbb}^{a^2Ab^4} \overbrace{bbb}^{b^2BC^6}$$

$$\rightarrow S \rightarrow B \rightarrow bBCCC \rightarrow bCCCCC \rightarrow \overbrace{bc}^{a^k} \overbrace{CCC}^{Ab^2k} \overbrace{CC}^{b^2BC^3}$$

$$\rightarrow bc \overbrace{ccc}^{a^k} \overbrace{CC}^{E^2b^2k} \rightarrow bccc \lambda C \rightarrow \overbrace{bccc}^{a^k} \overbrace{\lambda C}^{E^2b^2k} \overbrace{C}^{b^2C^2}$$

6(a). YES. Let $\varphi \in (A \cap B) \cdot C^*$. Then $\varphi = \alpha \cdot \gamma$ where $\alpha \in A \cap B$ and $\gamma \in C^*$. Since $\alpha \in A \cap B$, $\alpha \in A$ and $\alpha \in B$ and $\gamma \in C^*$. So $\varphi = \alpha \cdot \gamma \in A \cdot C^*$ bec. $\alpha \in A$ and $\gamma \in C^*$. Also $\varphi = \alpha \cdot \gamma \in B \cdot C^*$ because $\alpha \in B$ and $\gamma \in C^*$. Hence $\varphi \in A \cdot C^*$ and $\varphi \in B \cdot C^*$. So $\varphi \in (A \cdot C^*) \cap (B \cdot C^*)$. $\therefore (A \cap B) \cdot C^* \subseteq (A \cdot C^*) \cap (B \cdot C^*)$.

(b) YES. Let $\varphi \in A \cdot C - B \cdot C$. Then $\varphi \in A \cdot C$ and $\varphi \notin B \cdot C$.

Since $\varphi \in A \cdot C$, $\varphi = \alpha \cdot \gamma$ where $\alpha \in A$ and $\gamma \in C$. Also since $\varphi \notin B \cdot C$, then $\alpha \cdot \gamma \notin B \cdot C$. Hence $\alpha \notin B$ (because if $\alpha \in B$, then since $\gamma \in C$, $\alpha \cdot \gamma \in B \cdot C$ - a contradiction), so $\alpha \in A$ and $\alpha \notin B$ and $\gamma \in C$. $\therefore \alpha \in (A - B)$ and $\gamma \in C$. Hence $\alpha \cdot \gamma \in (A - B) \cdot C$. So $\varphi \in (A - B) \cdot C$ $\therefore A \cdot C - B \cdot C \subseteq (A - B) \cdot C$.

END