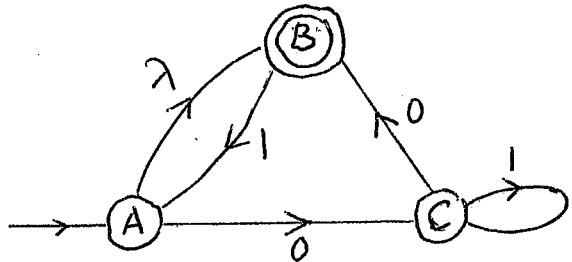


*Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.*

- (15) 1.(a) Define what is the *extended transition function* of a DFA  $M$ .  
 (b) Let  $M$  be the NFA on the right. Find a DFA,  $M_D$  which is *equivalent* to  $M$ .



- (15) 2. Find *regular expressions*,  $E_1$  and  $E_2$ , which describe the languages,  $L_1$  and  $L_2$ , below.  
 (a)  $L_1 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains both } bba \text{ and } aba \text{ as substrings}\}$   
 and indicate how  $aabaabbab$  is described by  $E_1$ .  
 (b)  $L_2 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains at most one occurrence of the string } 11\}$ .  
 and indicate how  $10011010$  is described by  $E_2$ .

- (20) 3. (a) Define what it means for two states  $p$  &  $q$  to be *indistinguishable* in a DFA,  $M$ .  
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent *reduced machine*,  $M_R$ .

	(A)	B	C	(D)	→(E)	F	(G)
0	C	D	G	C	C	A	F
1	F	F	C	B	G	B	E

- (15) 4. (a) Let  $f(\omega) = [2 \cdot n_b(\omega) - n_a(\omega) - 1] \pmod{4}$ . Find a DFA,  $M$  which accepts the language,  $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is } 1 \text{ or } 2 \pmod{4}\}$ .  
 (b) If  $\varphi = ababa$  find  $f(\varphi)$  & check that it agrees with your DFA with  $\varphi$  as input.

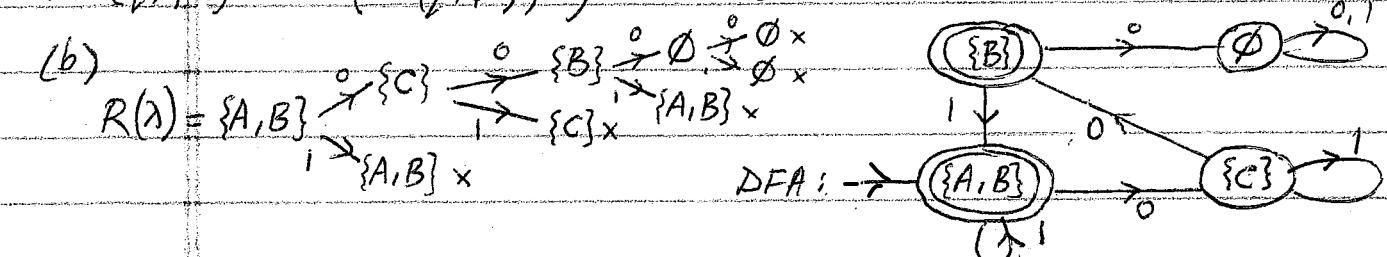
- (20) 5. (a) Find a *context-free grammar*  $G$  which generates the language  
 $L_5 = \{a^k b^n : n \geq 3k + 1, k \geq 0\} \cup \{b^k c^n : 0 \leq n \leq 2k+3, k \geq 0\}$ .  
 (b) Find *derivations* in  $G$  for each of the strings: (i)  $a^2 b^8$  and (ii)  $b^1 c^3$ .

- (15) 6. Let  $A$ ,  $B$ , and  $C$  be languages based on the *alphabet*  $\{a,b\}$ .  
 (a) Is it always true that  $(A \cdot B) - (A \cdot C) \subseteq A \cdot (B - C)$ ?  
 (b) Is it always true that  $A \cdot (B - C) \subseteq (A \cdot B) - (A \cdot C)$ ? (Justify your answers.)

1 (a) The extended transition function of DFA,  $\delta^*: Q \times T^* \rightarrow Q$  is defined recursively as follows. (i)  $\delta^*(q, \lambda) = q$  for any  $q \in Q$ , and (ii)  $\delta^*(q, \varphi; c) = \delta(\delta^*(q, \varphi), c)$  for each character  $c \in T$ .

(b)

$$R(\lambda) = \{A, B\} \xrightarrow{\overset{\circ}{\lambda}} \{C\} \times \{A, B\} \times \{A, B\} \times \dots$$



$$2(a) E_1 = (a+b)^*.(bba.(a+b)^*aba + aba.(a+b)^*bba + bbaba).(a+b)^*$$

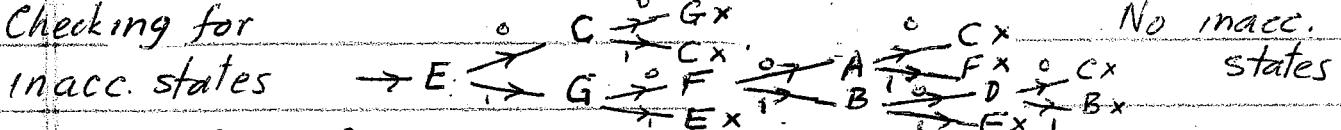
a. aba. a. bba. b The remaining a, a, and b from aba from bba came from three  $(a+b)^*$ 's

$$(b) E_2 = (0+10)^* + (0+10)^* \underline{1} + (0+10)^* \underline{11} \cdot (0+01)^*$$

10. 0. 11. 01. 0 The remaining two 0's came from  $(0+10)^*$  from 11 from  $(0+01)^*$  from  $(0+10)^*$  and  $(0+10)^*$ .

3 (a) Two states  $p$  &  $q$  are indistinguishable in a DFA  $M$ , if for each  $\varphi \in T^*$ ,  $\delta^*(q, \varphi) \in A(M) \Leftrightarrow \delta^*(p, \varphi) \in A(M)$ .

(b) Checking for



$$P_0: \{B, C, F\} \quad \{A, D, E, G\}$$

$$P_1: \{B, C, F\} \quad \{A, D\} \quad \{E, G\}$$

$$P_2: \{B, F\} \quad \{C\} \quad \{A, D\} \quad \{E, G\}$$

$$P_3: \{B, F\} \quad \{C\} \quad \{A, D\} \quad \{E\} \quad \{G\}$$

$$P_4: \{B, F\} \quad \{C\} \quad \{A, D\} \quad \{E\} \quad \{G\} = P_3 = \text{Final Partition}$$

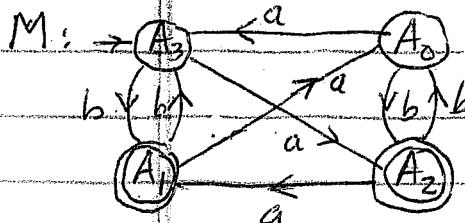
$M_R:$	$\rightarrow \{E\}$	$\{A, D\}$	$\{G\}$	$\{B, F\}$	$\{C\}$
0	$\{C\}$	$\{C\}$	$\{B, F\}$	$\{A, D\}$	$\{G\}$
1	$\{G\}$	$\{B, F\}$	$\{E\}$	$\{B, F\}$	$\{C\}$

4(a) Let  $A_i$  ( $i=0,1,2,3$ ) keep track of the output as the input is being processed.

Then  $A_1 \& A_2$  will be the accepting states & since  $f(\lambda) = 2(0) - 0 - 1 \equiv 3 \pmod{4}$ ,  $A_3$  will be the initial state. Now

$$f(wa) = 2n_b(wa) - n_a(wa) - 1 = [2n_b(w) - n_a(w) - 1] - 1 = f(w) - 1 \pmod{4}$$

$$f(wb) = 2n_b(wb) - n_a(wb) - 1 = [2n_b(w) - n_a(w) - 1] + 2 = f(w) + 2 \pmod{4}$$



Checking:  $f(ababa) = 2(2) - 3 - 1 \equiv 0 \pmod{4}$

$$\begin{matrix} a & b & a & b & a \\ A_3 & A_2 & A_0 & A_3 & A_1 & A_0 \end{matrix} \begin{matrix} \swarrow \\ \text{(halt)} \end{matrix}$$

$\checkmark$

5. (a)  $\rightarrow S, S \rightarrow A|B, A \rightarrow aAbbb|D, D \rightarrow Db|b$ .

$B \rightarrow bBCC|CCC, C \rightarrow c|a$ .

(b)  $\rightarrow S \Rightarrow A \Rightarrow aAbbb \Rightarrow aaAbbbbb \Rightarrow a^2Db^6 \Rightarrow a^2Db^6 \Rightarrow a^2.b.b^6 = a^2b^8$

$\rightarrow S \Rightarrow B \Rightarrow bBCC \Rightarrow b.CCC.CC \Rightarrow b.CCC.C.a \Rightarrow b.CCC.a.a$

$\Rightarrow b.cCC.a.a \Rightarrow b.ccC.a.a \Rightarrow b.ccc.a.a = b^3c^3$ .

6(a) YES. Let  $\varphi \in (A.B) - (A.C)$ . Then  $\varphi \in A.B$  and  $\varphi \notin A.C$ .

So  $\varphi = \alpha.\beta$  for some  $\alpha \in A$  and some  $\beta \in B$  and  $\alpha.\beta \notin A.C$

Now if  $\beta \in C$ , then we would have  $\varphi = \alpha.\beta \in A.C$  (because  $\alpha \in A$  &  $\beta \in C$ ) - and this would contradict the fact that  $\varphi \notin A.C$ . Hence  $\beta \notin C$ .

Since  $\beta \in B$  and  $\beta \notin C$ , we get that  $\beta \in (B-C)$ .

So  $\varphi = \alpha.\beta \in A.(B-C)$  [because  $\alpha \in A$  and  $\beta \in (B-C)$ ].

Hence  $\varphi \in (A.B) - (A.C) \Rightarrow \varphi \in A.(B-C)$ .  $\therefore (A.B) - (A.C) \subseteq A.(B-C)$

(b) NO. Let  $A = \{b, ba\}$ ,  $B = \{b\}$ , and  $C = \{ab\}$ . Then

$$A.(B-C) = \{b, ba\}.(\{b\} - \{ab\}) = \{b, ba\}.\{b\} = \{bb, bab\}$$

$$(A.B) - (A.C) = (\{b, ba\}.\{b\}) - (\{b, ba\}.\{ab\}) = \{bb, bab\} - \{bab, baab\} = \{bb\}$$

So  $A.(B-C) \neq (A.B) - (A.C)$  because  $bab \in A.(B-C)$  but

but  $bab \notin (A.B) - (A.C)$ . Hence it is not always true that

$$A.(B-C) \subseteq (A.B) - (A.C)$$

END