

SECTION 1.2 (ANSWERS OR HINTS) p. 28

#5 (a)  $a.b.a.a.b.a.a.b.b.a.a$ ,  $a.a.a.a.b.a.a.b.b.a.a$ ,  $b.a.a.a.a.b.b.a.a$   $\in L^*$   
 $a.b.a.a.b.b.a.a.b.b$  is not in  $L^4$  but the next two are in  $L^4$

(b)  $b.a.a.a.a.b.b.a.a.b$  is not in  $L^*$  nor  $L^4$ .

#6.  $\bar{L} = \{\lambda, a, b, ab, ba\} \cup \{\varphi \in \{a, b\}^*: |\varphi| \geq 3\}$ .

#25 (a) If  $L = \{a^n b^{n+1} : n \geq 0\}$  then  $L \neq L^*$  because  
 $\lambda \notin L$  but  $\lambda \in L^*$

(b) If  $L = \{w : n_a(w) = n_b(w)\}$  then  $L = L^*$ . Prove this.

#7 Hint:  $L \cup \bar{L} = V^*$  and  $V^*$  is infinite.

#8 No. No matter what  $L$  is, we will have  
 $\lambda \in L^*$ . So  $\lambda \notin \bar{L}^*$ . But  $\lambda \in (\bar{L})^*$ . So  
we can never have  $\bar{L}^* = (\bar{L})^*$ .

#11 (a) TRUE (b) TRUE

#14 (a)  $S \rightarrow AA, A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$  (exactly 2 a's)

(b)  $S \rightarrow AA, A \rightarrow AA, A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$  ( $\geq 2$  a's)

(c)  $S \rightarrow AAA, A \rightarrow \lambda$  (no more than 3 a's)  
 $A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$

(d)  $S \rightarrow AAA, A \rightarrow AA$  (at least 3 a's)  
 $A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$

#15  $L(G) = \{(aa'b)^n : n \geq 0\}$

#16.  $L(G) = \emptyset$

Extra Prob.  $L = \{w : n_a(w) = n_b(w)\}$ . Since  $L^* = \bigcup_{k=0}^{\infty} L^k$ ,  $L \subseteq L^*$ .

Now suppose  $\varphi \in L^*$ . Then  $\varphi = \varphi_1 \varphi_2 \dots \varphi_k$  where

each  $\varphi_i \in L$ . So  $n_a(\varphi) = n_a(\varphi_1) + n_a(\varphi_2) + \dots + n_a(\varphi_k)$

$= n_b(\varphi_1) + n_b(\varphi_2) + \dots + n_b(\varphi_k) = n_b(\varphi)$  bec. each  $\varphi_i \in L$ .

So  $\varphi \in L$  bec.  $n_a(\varphi) = n_b(\varphi)$ . So  $\varphi \in L$ . Hence  $L^* = L$ .

Extra Prob.: If  $L = \{w \in \{a, b\}^*: n_a(w) = n_b(w)\}$ , show that  $L^* = L$ .

SECTION 1.2 p.29

(2)

- 17 (a)  $S \rightarrow aSb / Sb / b$   $L_1 = \{a^n b^m : n \geq 0, m > n\}$
- (b)  $S \rightarrow aSbb / \lambda$   $L_2 = \{a^n b^{2n} : n \geq 0\}$
- (c)  $S \rightarrow aaA, A \rightarrow aAb / ab$   $L_3 = \{a^{n+2} b^n : n \geq 2\}$
- (d)  $S \rightarrow aaaA, A \rightarrow aAb / \lambda$   $L_4 = \{a^n b^{n-3} : n \geq 3\}$
- (e)  $S \rightarrow AB, A \rightarrow aAb / Ab / b, B \rightarrow aBbb / \lambda$ ,  $L_1 \cup L_2$
- (f)  $S \rightarrow A/B, A \rightarrow aAb / Ab / b, B \rightarrow aBbb / \lambda$ ,  $L_1 \cup L_2$
- (g)  $S \rightarrow AAA, A \rightarrow aAb / Ab / b$   $L_1^3$
- (h)  $S \rightarrow SA / \lambda, A \rightarrow aAb / Ab / b$   $L_8 = L_1^*$

- 18 (a)  $S \rightarrow SS / aaa / \lambda$   $L = \{w : |w| \pmod 3 = 0\}$
- (b)  $S \rightarrow Saaa / aa / a$   $L = \{w : |w| \pmod 3 > 0\}$
- (c)  $S \rightarrow Sa^6 / a^5 / a^4 / a^3 / a^2$   $L = \{w : |w| \pmod 3 + |w| \pmod 2 = 0\}$
- (d)  $S \rightarrow Sa^6 / a^5 / a^4 / a^2 / a / \lambda$   $L = \{w : |w| \pmod 3 \geq |w| \pmod 2\}$

19.  $S \rightarrow aSa / bSb / aa / bb$

20.  $L(G) = \{\varphi a \varphi^T : \varphi \in \{a, b\}^*\}$  where  $\varphi^T = \varphi$  with all a's replaced by b  
and all b's replaced by a.

- 5th ed. (18). (a)  $S \rightarrow aA / AS, A \rightarrow AA / aAb / bAa / \lambda$

- (b)  $S \rightarrow aS / AS / aA, A \rightarrow AA / aAb / bAa / \lambda$

- (c)  $S \rightarrow abSa / aaSb / bSaa / SS / \lambda$

- 5th ed.) (21). No. If  $G_1 := S \rightarrow aSb / ab / \lambda$  and

$G_2 := S \rightarrow aAb / ab, A \rightarrow aAb / \lambda$  then  $\lambda \in L(G_1)$

but  $\lambda \notin L(G_2)$ . Actually  $L(G_1) = \{a^n b^n : n \geq 0\}$   
and  $L(G_2) = \{a^n b^n : n \geq 1\}$

22. The only new production is  $S \rightarrow SSS$ , but we can simulate this from Ex. 1.13 by  $S \Rightarrow SS \Rightarrow SSS$ .

24. Hint: aa can be generated by first grammar but not by second one

SECTION 2.1 p. 49

3

- #1 0001 and 01001 will be accepted  
0000110 will not be accepted

- #4 (a)  [all strings with exactly one a]

- (6)  [all strings with at least one a]

- (c) 

[all strings with no more than 3 a's]

- (d)

#9 Let  $\varphi \in \bar{L}$ . Then  $\varphi$  will be rejected by  $M$ . So when we input  $\varphi$  in  $M$  we will end up at  $q_0$ ,  $q_1$ , or  $q_2$ . Since these are accepting states in  $M^c$ ,  $\varphi$  will be accepted by  $M^c$ . So  $\varphi \in L(M^c)$ .

Now let  $\varphi \in L(M^c)$ . Then  $\varphi$  will be accepted by  $M^c$ . So when we enter  $\varphi$  in  $M^c$ , we will end up at  $q_0$ ,  $q_1$ , or  $q_2$ . But these are rejecting states in  $M$ . So  $\varphi$  will be rejected by  $M$ . So  $\varphi \notin L(M) = L$ . So  $\varphi \in \bar{L}$ . Hence  $\bar{L} = L(M^c)$ .

(4)

## SECTION 2.1 p. 49

#10 Let  $\varphi \in L(M)$ . Then  $\delta^*(q_0, \varphi) \in F$ . Now

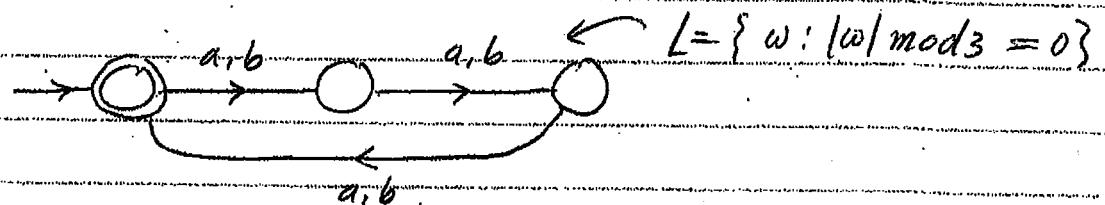
$$\varphi \in \overline{L(M)} \Leftrightarrow \varphi \notin L(M)$$

$$\Leftrightarrow \delta^*(q_0, \varphi) \notin F$$

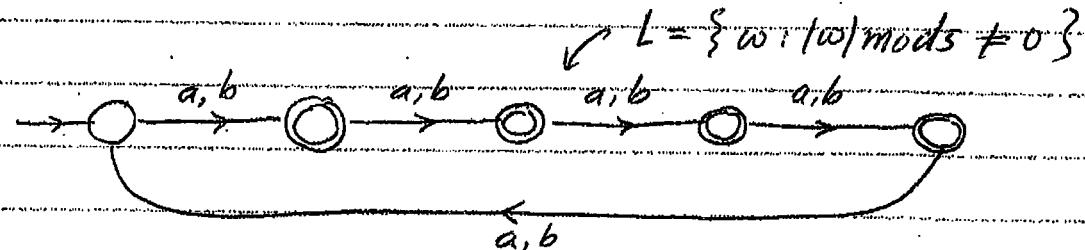
$$\Leftrightarrow \delta^*(q_0, \varphi) \in Q - F$$

$$\Leftrightarrow \varphi \in \overline{L(\hat{M})}$$

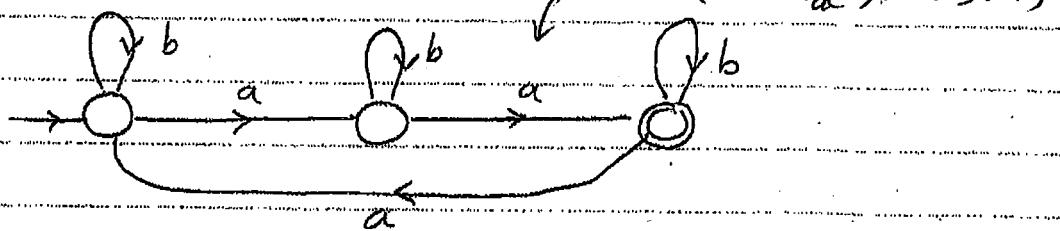
#7 (a)



(b)



(c)



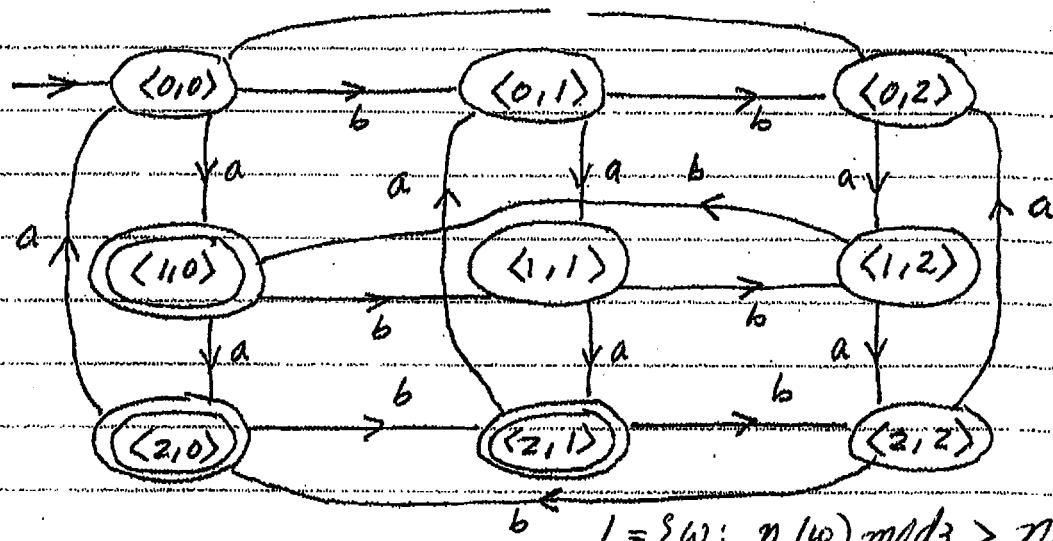
(d) Use the 9 states  $\{(0,0), (0,1), (0,2), (1,0), \dots, (2,2)\}$   
 $= \{0,1,2\} \times \{0,1,2\}$  to keep track of  $n_a(w)$  &  
 $n_b(w)$ . The first component will be  $n_a(w)$   
(mod 3) and the second will be  $n_b(w)$  (mod 3).

$\langle 0,0 \rangle$  will be the starting state b.c.  $n_a(\lambda) = 0 = n_b(\lambda)$ .  
 $\langle 1,0 \rangle, \langle 2,0 \rangle$  &  $\langle 2,1 \rangle$  will be accepting states b.c.  
 $n_a(w) > n_b(w)$  (mod 3) for these states.

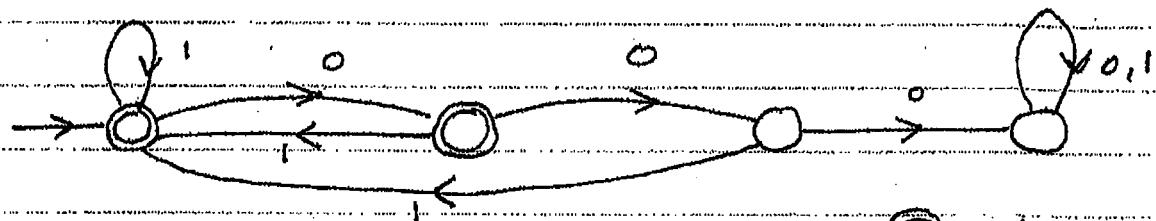
SECTION 2.1 p. 49'

(5)

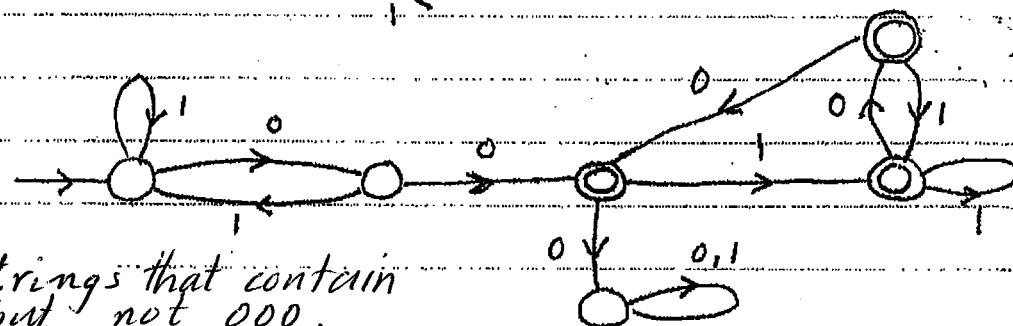
7(d)



11(a)

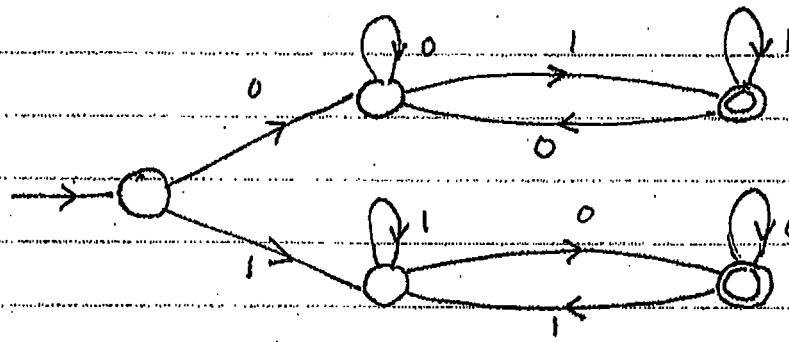


(b)



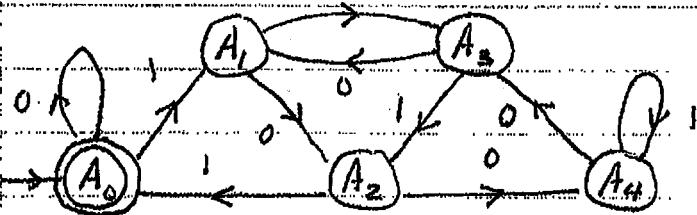
All strings that contain  
00 but not 000.

(c)



The left-most symbol differs from the right-most symbol.

12



Let  $A_i$  keep track of the binary value of  $\varphi \pmod{3}$  where  $\varphi$  = input string so far. Initially  $\varphi = \alpha$ .

(6)

## SECTION 2.1 p. 50

#12 Example:

$$\begin{array}{ccccccc} 1 & 1 & 1 & 0 & 0 & 1 \\ A_0 & A_1 & A_3 & A_2 & A_4 & A_3 & A_2 \end{array}$$

Successive inputs:  $(\lambda)_2 = 0 \rightarrow A_0$ 

$$(\lambda)_2 = 1 \rightarrow A_1 \quad (11)_2 = 3 \rightarrow A_3 \quad 7 = (111)_2 \rightarrow A_2$$

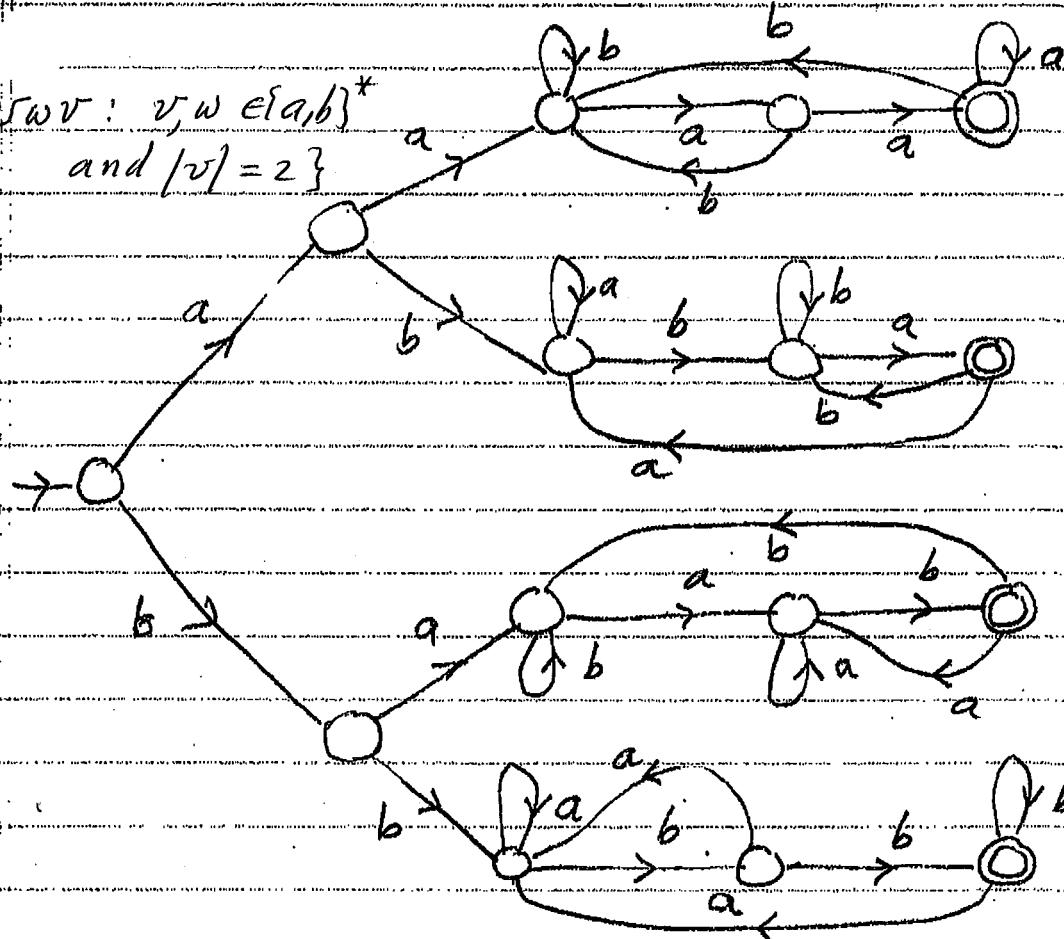
$$1110 = 14 \rightarrow A_4 \quad (11100)_2 = 28 \rightarrow A_3 \quad 111001 = 57 \rightarrow A_2$$

$$\text{Note: } (\varphi 0)_2 = 2(\varphi)_2 + 0$$

$$(\varphi 1)_2 = 2(\varphi)_2 + 1$$

#13

$$L = \{uvv : v, w \in \{a, b\}^* \text{ and } |v| = 2\}$$

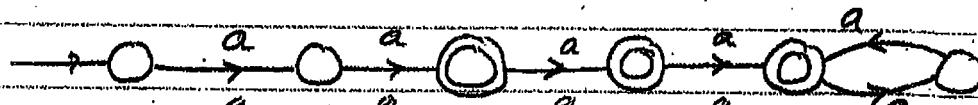


$$\begin{array}{l} \#14 \rightarrow \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \rightarrow L = \{a^n : n \geq 3\} \\ \#15 \rightarrow \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \text{---} \xrightarrow{a} \rightarrow L = \{a^n : n \neq 3\} \end{array}$$

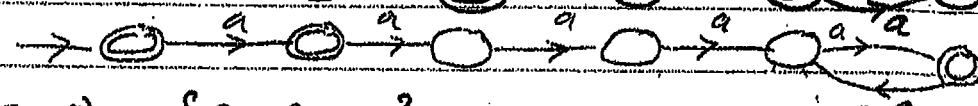
(7)

## SECTION 2.2 p.57

#3



#4



#6

$$\delta^*(q_0, a) = \{q_0, q_1, q_2\}$$

$$\delta^*(q_1, a) = \{q_0, q_1, q_2\}$$

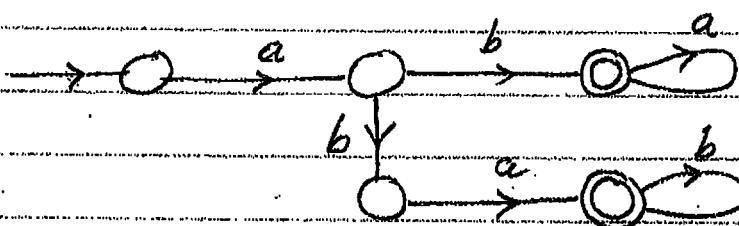
#5

$$\delta^*(q_0, 1010) = \{q_0, q_2\} \quad \delta^*(q_0, 1011) = q_2$$

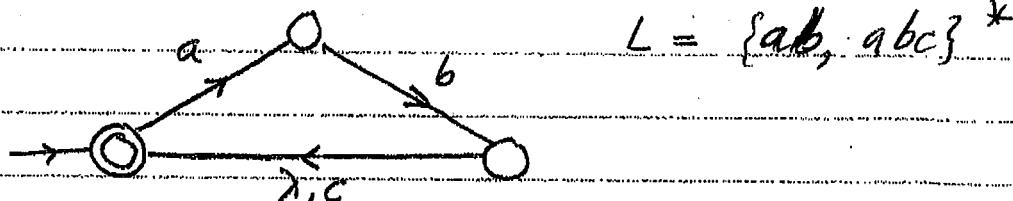
$$\delta^*(q_1, 00) = \emptyset$$

$$\delta^*(q_1, 01) = \emptyset.$$

#8

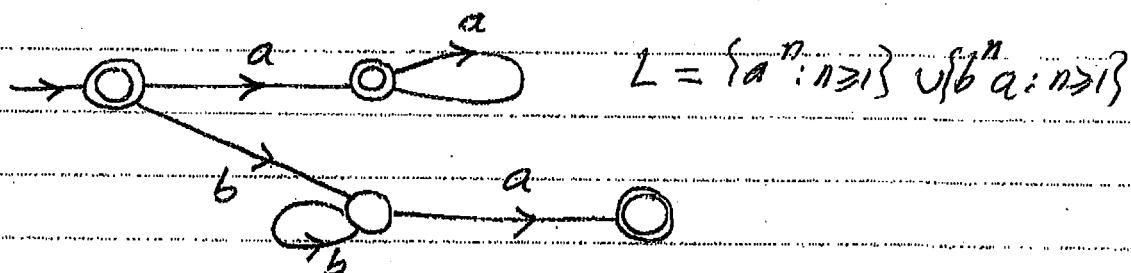


#9.



$$L = \{ab, abc\}^*$$

#12



$$L = \{a^n : n \geq 1\} \cup \{b^n a : n \geq 1\}$$

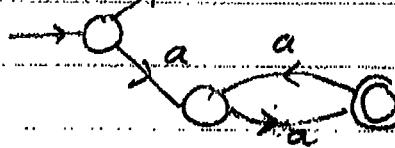
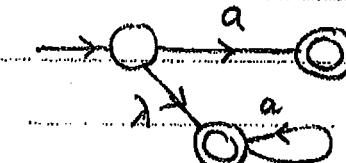
#13

01001, 000

#15.



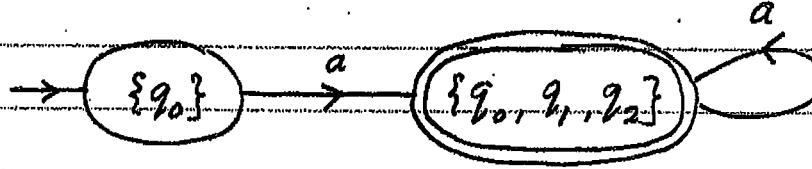
#17



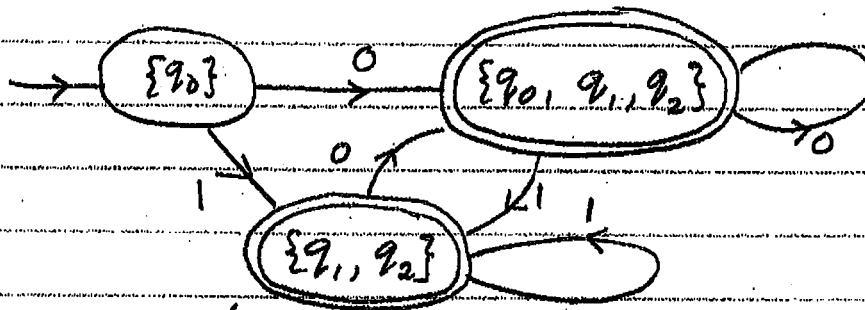
(8)

## SECTION 2.3 p. 624

#1

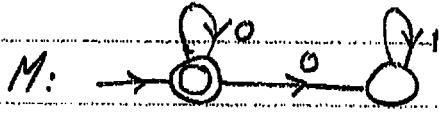


#3



#7 Yes; Hint:  $L(M) = \{q \in \Sigma^*: \delta^*(q_0, q) \cap F \neq \emptyset\}$   
 $\overline{L(M)} = \{q \in \Sigma^*: \delta^*(q_0, q) \cap F = \emptyset\}$

#8 No. The complement of  $L(M)$  can include strings which are rejected by  $M$ , not because they drive  $M$  into a non-accepting state, but because  $M$  lacks transitions to process them.



$$\Sigma = \{0, 1\}, \quad 1 \notin L(M)$$

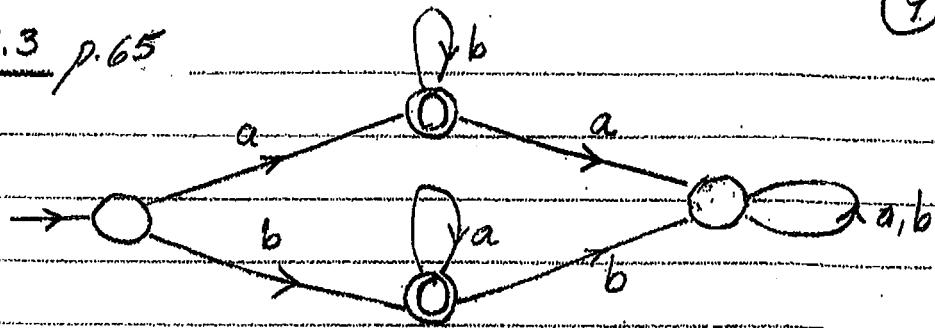
So  $1 \in \overline{L(M)}$  but  $1 \notin \{w \in \Sigma^*: \delta^*(q_0, w) \cap Q - F \neq \emptyset\}$   
because  $\delta^*(q_0, 1) = \emptyset$ .

#9 (a) Hint: Let  $M = \langle Q, \Sigma, \delta, q_0, F \rangle$ . Change  $M$  to  $M'$  by adding  $\lambda$ -transitions from all accepting states in  $M$  to a new accepting state in  $M'$ , and make all the accepting states in  $M$  non-accepting in  $M'$ . Then check that  $L(M') = L(M)$ .

SECTION 2.3 p.65

9

#9 (b) No.

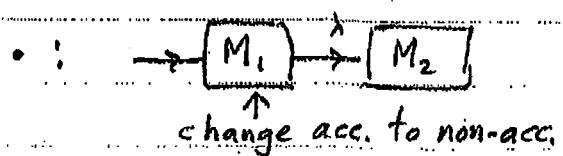
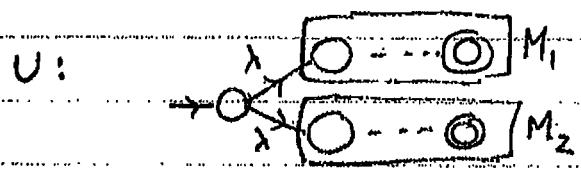
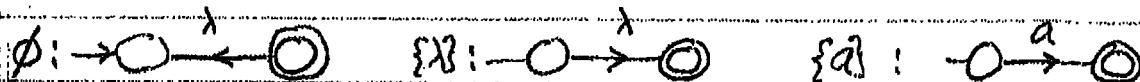


#12 Yes. First introduce a new initial state  $q_0'$  and add  $\lambda$ -transitions from it to all the initial states of  $M$  to get a new machine  $M'$ . In  $M'$ ,  $q_0'$  will be the only initial state. Now  $M'$  will be an nfa, so we can convert it into a dfa  $M''$ . The dfa  $M''$  will have only one initial state and will be equivalent to our original  $M$  which had multiple initial states.

#13(a) One way is to just show that every finite language can be described by a reg. expression.

$$\text{Ex. } \{ab, baa, abab\} = (ab + baa + abab)$$

(b) We can also show that we can find an nfa which accepts any finite language by starting with simple nfa's & using union & concatenation.



(10)

SECTION 2.4 p. 72  
non-accepting

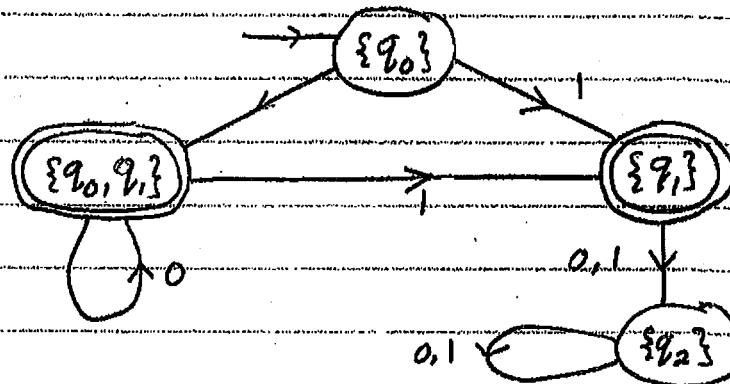
accepting states

#2  $P_0 : \{\{q_0\}, \{q_2\}, \emptyset\}$ ,  $\{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

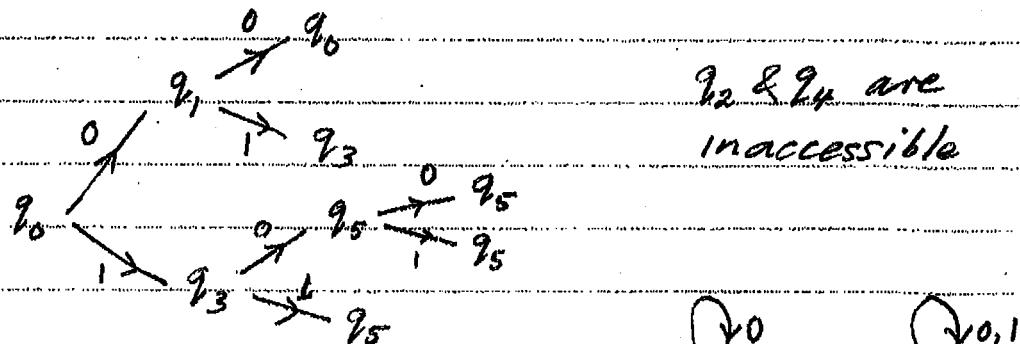
$P_1 : \{\{q_0\}\}, \{\{q_2\}, \emptyset\}, \{\{q_1\}, \{q_1, q_2\}\}, \{\{q_0, q_1\}, \{q_0, q_1, q_2\}\}$

$P_2 : = P_1$

Reduced dfa:



5th ed.) #4. (a)



(b)  $P_0 : \{q_0, q_1\} \{q_3, q_5\}$   $M^P : \rightarrow q_1 \rightarrow q_3$   
 $P_1 : \{q_0, q_1\} \{q_3, q_5\} = P_0$

#6 The conjecture is true. Suppose not. Then we can find a minimal DFA  $M$  for  $L$  such that  $\hat{M}$  is not minimal for  $\bar{L}$ . Now minimize  $\hat{M}$  to get a smaller DFA  $N$  for  $\bar{L}$ . By switching accepting & non-acc. states in  $N$ , we will get a DFA  $\hat{N}$  for  $\bar{L} = L$ , contradicting the minimality of  $M$ . So result is true.

(11)

## SECTION 3.1 p. 78

#2.  $a, ab, ba, bb, aab, aba, abb, baa, bab, bba, bbb$ #1.  $aaa.b\bar{b}, \underline{aa}.\underline{bb}.1, abbaa, bb.\underline{aa}, a.\underline{bb}.bb, \underline{b}abb, b\bar{b}.bb.a$ 

#5. Yes.

#6 (a) see page 12 for detailed solution

(b)  $(1+01)^*(0+1^*)\cdot 1^*$  a second reg. expr(c)  $(1+01)^*(0+\lambda)\cdot \lambda$  a third reg. expr#7  $\underline{aaa}\underline{a}^*. (\underline{bb})^*\underline{b}$ #8  $a(\underline{aa})^*(\underline{bb})^* + (\underline{aa})^*\underline{b}(\underline{bb})^*$ #9 (a)  $\underline{aaaaa}\underline{a}^*(\lambda + \underline{b} + \underline{bb} + \underline{bbb})$ (b)  $(\lambda + a + aa + aaa) \cdot (\lambda + \underline{b} + \underline{bb} + \underline{bbb} + \underline{bbb})$ #10 (a)  $\{1\}$  (b)  $\emptyset$ 

#11 Set of all strings that consists of a "b" surrounded by an even no. of a's on both sides or an odd number of a's on both sides.

$$\{a^m b a^n : m-n \equiv 0 \pmod{2}\}$$

#12  $(ba+a)^*.b.(b+a)^*$ #13 We split  $L = \{a^n b^m : n, m \geq 3, n \geq 1 \text{ & } m \geq 1\}$  into three pieces according to the conditions $n \geq 3, m=1$  To get  $\{a^n b : n \geq 3\}$  $n \geq 2, m=2$  "  $\{a^n bb : n \geq 2\}$  $n \geq 1, m \geq 3$  "  $\{a^n b^m : n \geq 1, m \geq 3\}$ Then answer is  $\underline{aaa}\underline{a}^*b + \underline{aaa}^*bb + \underline{aa}^*bb\underline{bb}^*$

(1)

SECTION 3.1 (Number 3 redone) p. 76

#3 (a) Let  $R_1 = (\underline{1+01})^* \cdot (\underline{0+1}^*)$  and  
 $R_2 = (\underline{1+01})^* \cdot (\underline{0+1})$   
be the expression from Example 3.6.

Clearly  $L(R_2) \subseteq L(R_1)$  because  $0+\lambda \subseteq 0+1^*$   
 $(\lambda \in 1^*)$ .

Now let  $\varphi \in L(R_1)$ . Then

$$\varphi = \alpha \cdot \beta \quad \text{where } \alpha \in L((\underline{1+01})^*) \text{ and} \\ \beta = 0 \text{ or } \beta \in L(1^*)$$

But if  $\beta = 0$ , then  $\varphi \in L(R_2) = L((\underline{1+01})^* \cdot (0+1))$

And if  $\beta \in L(1^*)$ , then  $\beta = 1^n$  for some  
 $n \geq 0$ . So

$$\begin{aligned} \varphi &= \alpha \cdot 1^n \quad \text{with } \alpha \in L((\underline{1+01})^*) \\ &= 1's \& (01)'s \text{ followed by } n \text{ 1's} \\ &\in L(\underline{1+01})^* = L((\underline{1+01})^* \cdot \lambda) \\ &\in L(\underline{1+01})^* \cdot (\underline{0+1}) \end{aligned}$$

$\therefore L(R_1) \subseteq L(R_2)$ . Hence  $L(R_1) = L(R_2)$

(b)  $(01+1)^* \cdot (\lambda+0)$  and  $(1+01)^* \cdot (\lambda+0+1)$   
are two other expressions which are  
equivalent to  $(\underline{1+01})(\underline{0+1}^*)$ .

#14  $L = \underline{a} \underline{b} \underline{b} \underline{b} \underline{b}^* (\underline{a+b}) (\underline{a+b})^*$

#15  $L =$  set of all strings which consists of an even no. of a's followed by an odd no. of b's  
 $= \{a^{2n} b^{2m+1} : n \geq 0, m \geq 0\}$

$L^c =$  set of all strings of the form  $a^n b^m$  with n odd or m even, or of strings with a "b" in front of an "a"  
 $= \{a^{2n+1} b^m : n \geq 0, m \geq 0\} \cup \{a^n b^{2m} : n \geq 0, m \geq 0\}$   
 $\cup \{\text{anything. } ba. \text{ anything}\}$

An expression for  $L^c$  is now easily seen to be  
 $a(\underline{aa})^* \underline{b}^* + \underline{a}^* (\underline{bb})^* + (\underline{a+b})^* \underline{ba} \cdot (\underline{a+b})^*$

#16  $\underline{aa}(\underline{a+b})\underline{aa} + \underline{ab}(\underline{a+b})\underline{ab} + \underline{ba}(\underline{a+b})\underline{ba} + \underline{bb}(\underline{a+b})\underline{bb}$ .

#17 A silly question. The answer is  $(\underline{a+b})^*$ .

#18  $(\underline{1+0!})^* \underline{00} \cdot (\underline{1+10})^*$

#19 (a)  $(b+c)^* \underline{a} (\underline{b+c})^* \cdot \underline{a} (\underline{b+c})^*$

(b) Look at the four cases: no a's, one a, two a's and three a's. With these cases we get:

$$(b+c)^* + (b+c)^* \cdot a \cdot (b+c)^* + (b+c)^* \cdot a \cdot (b+c)^* \cdot a \cdot (b+c)^*$$

$$+ (b+c)^* \cdot a \cdot (b+c)^* \cdot a \cdot (b+c)^* \cdot a \cdot (b+c)^*$$

(c) Look at six cases: ...a...b...c..., ...a...c...b..., ...b...a...c..., ...b...c...a..., ...c...a...b..., ...c...b...a...  
Now insert  $(a+b+c)^*$  for the dots.

# 20 (a)  $(0+1)^* \cdot 10$  (f)  $(0+\lambda)(1+00+000)(0+\lambda)^*$

(b)  $(\lambda+0+1) + (0+1)^*(00+01+11)$

(c)  $(1^*01^*01^*)^* + 1^*$  is one answer

$(1+01^*0)^*$  is another answer

5th ed. { (d)  $(0+1)^*(000+00(0+1)^*00)(0+1)^*$

# 17 { (e)  $(01+1)^*(\lambda+0+00+000+00(10+1)^*100)(10+1)^*$

# 22 (a) Clearly  $L(r_i^*) \subseteq L((r_i^*)^*)$ . Now

let  $\varphi \in L((r_i^*)^*)$ . Then

$\varphi = \alpha$  string of strings from  $r_i^*$

= a string of strings of strings from  $r_i$

= a string of strings from  $r_i$

$$\in L(r_i^*)$$

$$\therefore L((r_i^*)^*) \subseteq L(r_i^*)$$

Thus  $L((r_i^*)^*) = L(r_i^*)$  i.e.  $(r_i^*)^* \equiv r_i^*$

(b) Again clearly  $L((r_1+r_2)^*) \subseteq L(r_1^*(r_1+r_2)^*)$  because  $\lambda \in L(r_1^*)$ .

Now let  $\varphi \in L(r_1^*(r_1+r_2)^*)$ . Then

$\varphi = \alpha \cdot \beta$  with  $\alpha \in L(r_1^*)$  &  $\beta \in L((r_1+r_2)^*)$

=  $\alpha_1 \alpha_2 \dots \alpha_m \cdot \beta_1 \beta_2 \dots \beta_n$  with the  $\alpha_i$ 's in  $L(r_1)$  and  $\beta_j$ 's in  $L(r_1+r_2)$

=  $\alpha_1 \alpha_2 \dots \alpha_m \beta_1 \dots \beta_n$  with the  $\alpha_i$ 's and  $\beta_j$ 's in  $L(r_1+r_2)$

$$\in L((r_1+r_2)^*)$$

$$\therefore L(r_1^*(r_1+r_2)^*) \subseteq L((r_1+r_2)^*)$$

So  $L(r_1^*(r_1+r_2)^*) = L((r_1+r_2)^*)$  i.e.  $r_1^*(r_1+r_2)^* \equiv (r_1+r_2)^*$ .

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#22 (c) First observe that  $L(r_1^* r_2^*) \supseteq L(r_1 + r_2)$   
 b/c.  $\lambda \in L(r_1^*)$  and  $\lambda \in L(r_2^*)$ . So  
 $L((r_1^* r_2^*)^*) \supseteq L((r_1 + r_2)^*)$

Now let  $\varphi \in L((r_1^* r_2^*)^*)$ . Then  
 $\varphi$  = a string of things which are made  
 of a string from  $r_1^*$  followed by  
 a string from  $r_2^*$   
 = a string of things which are made  
 up of strings of things in  $r_1$  followed  
 by strings of things in  $r_2$ .  
 = a string of things from either  $r_1$   
 or  $r_2$   
 $\in L((r_1 + r_2)^*)$ .

So  $L((r_1^* r_2^*)^*) \subseteq L((r_1 + r_2)^*)$ . Thus

$$L((r_1^* r_2^*)^*) = L((r_1 + r_2)^*) \text{ i.e. } (r_1 + r_2)^* \equiv (r_1^* r_2^*)^*$$

d) false.  $(\underline{a} \cdot \underline{b})^* \neq \underline{a}^* \cdot \underline{b}^*$  because  $abab \in (\underline{a} \cdot \underline{b})^*$   
 but  $abab \notin \underline{a}^* \cdot \underline{b}^*$ .

#21. (a)  $\underbrace{[(\underline{a} + \underline{b})(\underline{a} + \underline{b})(\underline{a} + \underline{b})]}_{\substack{\text{strings with lengths} \\ \text{that are a mult. of 3}}}^* \cdot \underbrace{[(\underline{a} + \underline{b}) + (\underline{a} + \underline{b})(\underline{a} + \underline{b})]}_{\substack{\text{length 1} \\ \text{strings} \\ \text{length 2} \\ \text{strings}}}$

(b)  $\underbrace{(\underline{b} + [\underline{a} \cdot \underline{b}^* \cdot \underline{a} \cdot \underline{b}^* \cdot \underline{a}])^*}_{\text{all strings with } (3k) \text{ a's}}, \underbrace{(\underline{b}^* \cdot \underline{a} \cdot \underline{b}^* \cdot \underline{a} \cdot \underline{b}^* \cdot \underline{a} \cdot \underline{b}^*)^*}_{\text{another answer}}$

(c)  $\underbrace{(\underline{b} + [\underline{a} \cdot \underline{b}^* \cdot \underline{a} \cdot \underline{b}^* \cdot \underline{a} \cdot \underline{b}^* \cdot \underline{a} \cdot \underline{b}^* \cdot \underline{a}])^*}_{\text{strings with } 5k \text{ a's}} \cdot \underbrace{[\underline{a} + \underline{a} \underline{b}^* \underline{a} + \underline{a} \underline{b}^* \underline{a} \underline{b}^* \underline{a} + \underline{a} \underline{b}^* \underline{a} \underline{b}^* \underline{a} \underline{b}^* \underline{a}]}_{\substack{\text{strings with } 1, 2, 3 \text{ or } 4 \text{ a's}}} \cdot \underline{b}^*$

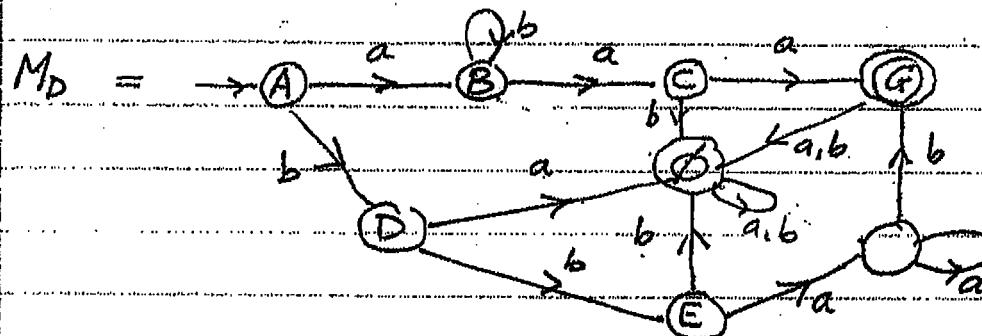
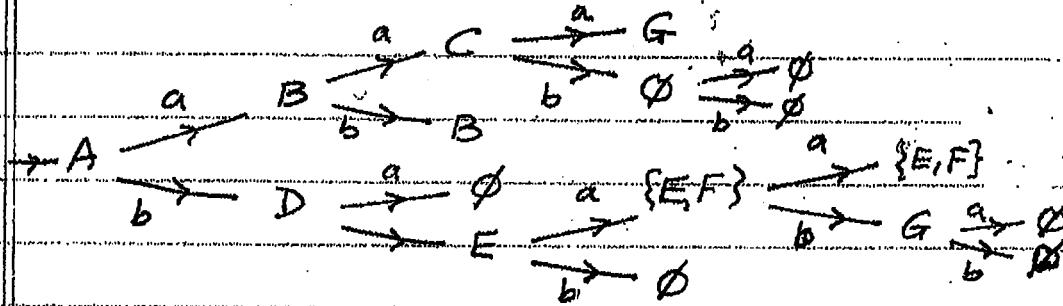
SECTION 3.2 p. 89

#3  $M =$

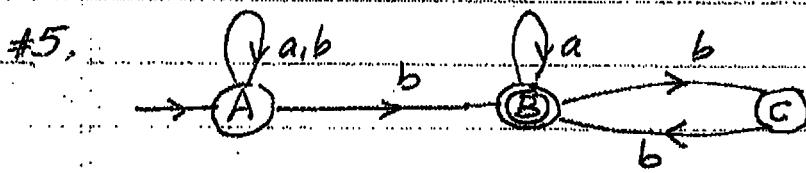
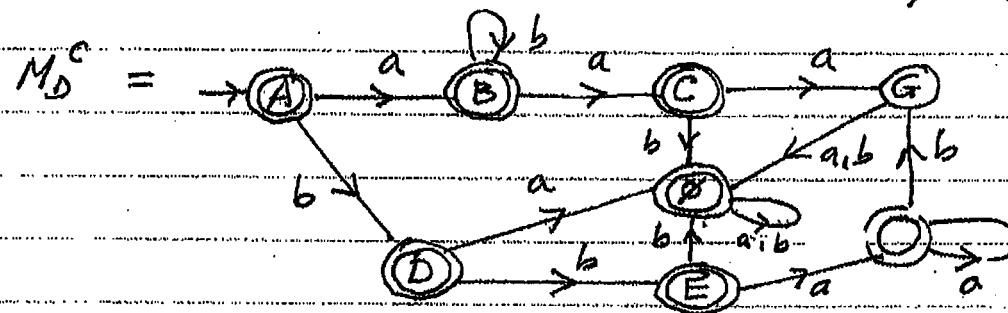
```

graph LR
    A((A)) -- a --> B((B))
    A -- b --> D((D))
    B -- a --> C((C))
    B -- b --> E((E))
    C -- a --> G((G))
    D -- a --> E
    D -- b --> F((F))
    E -- a --> F
    F -- b --> G
  
```

#4 (a) First convert M into a dfa M<sub>d</sub>



(b) Then switch accepting & non-accepting states

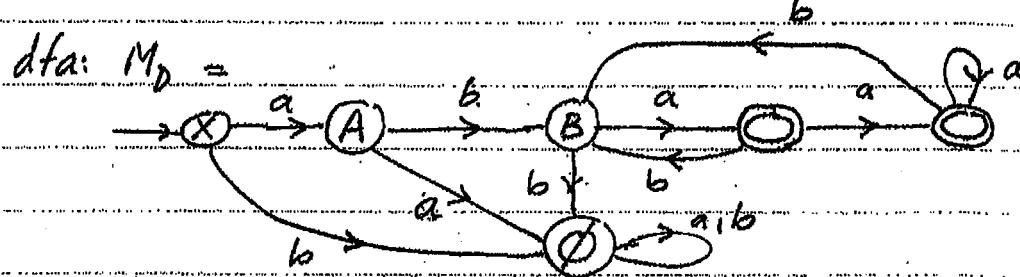
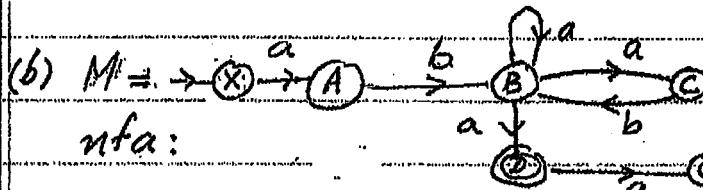
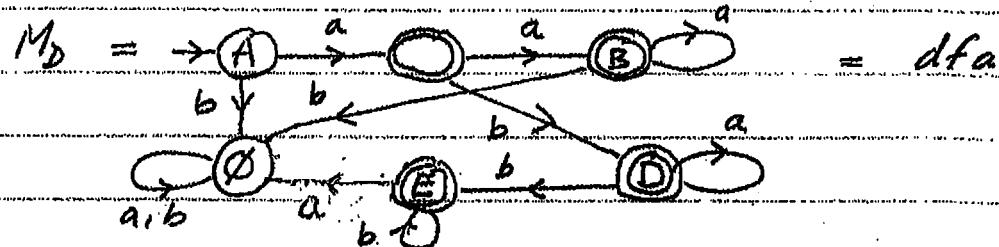
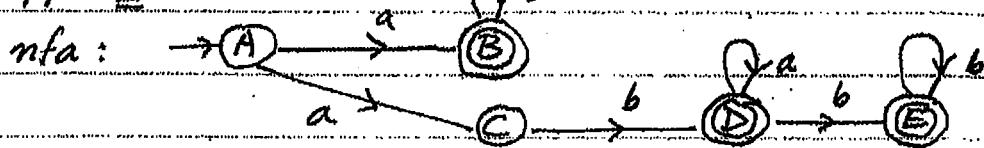


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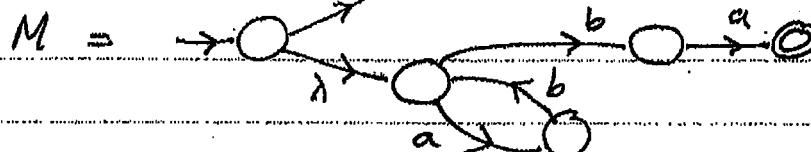
(17)

#6. First find an nfa and then convert it into a dfa

(a)  $M =$

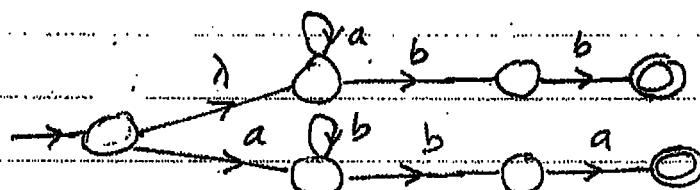


#7 (a) nfa,



Now convert  $M$  into a dfa  $M_D$ .

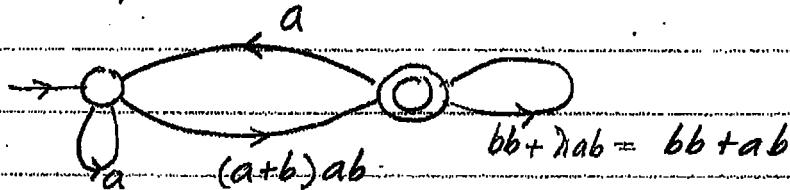
#9 nfa  $M =$



Now convert  $M$  into a dfa  $M_D$  and then minimize  $M_D$  by using the Partition Algorithm.

## SECTION 3.2 p. 90

#10 (a)

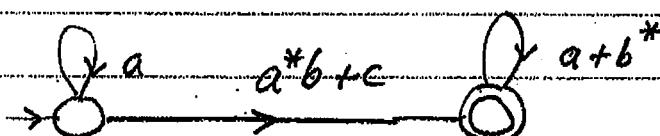


(b)

$$a^* \cdot (a+b)ab \cdot ((bb+ab) + a \cdot a^*(a+b)ab)$$

$\uparrow r_1^*$        $\uparrow r_2$        $\uparrow r_4$        $\uparrow r_3$        $\uparrow r_1^* \cdot r_2$

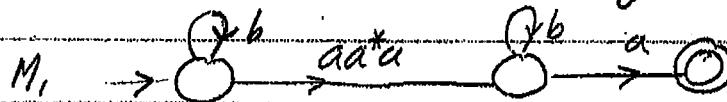
#11



$$L(M) = a^* \cdot \underbrace{(a^*b+c)}_{r_1^*} \cdot \underbrace{(a+b^*)^*}_{r_2 \sim r_4}$$

$\uparrow r_1^*$        $\uparrow r_2$        $\uparrow r_4$

#12 (a) Since this is an easy example, you can instantly see that  $L(M) = b^*aa^*aba^*$ . But let's see how the algorithm proceeds.



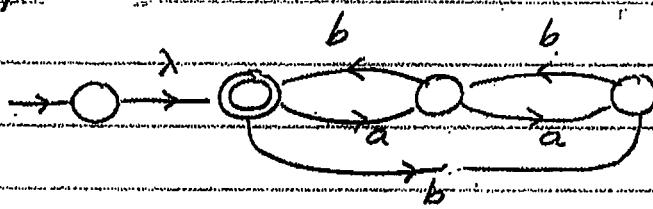
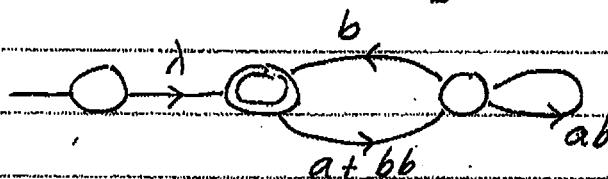
$$L(M) = b^* \underbrace{aa^*ab}_r \cdot (\emptyset)^* \leftarrow r^* r_2 (r_4 + r_3 r_1^* r_2)^*$$

$\uparrow r_1^*$        $\uparrow r_2$        $\uparrow r_4 + r_3 r_1^* r_2$

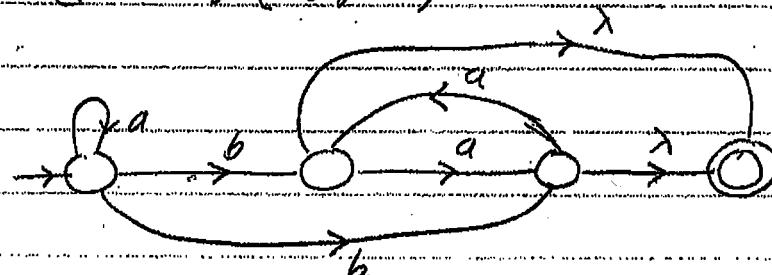
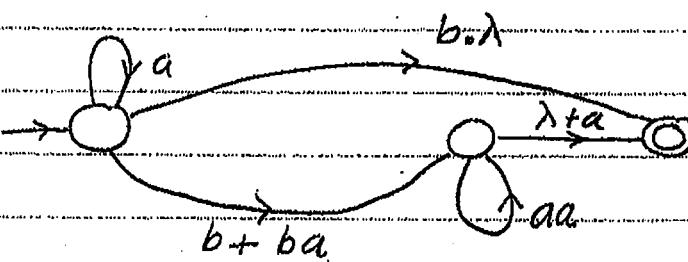
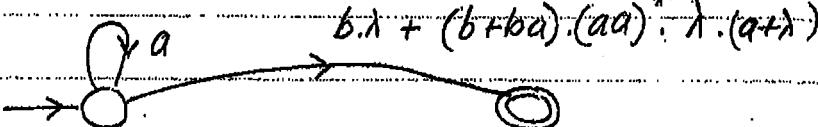
$\emptyset = \emptyset$   
 b.c.  $r_3 = \emptyset$

$$= \underline{b^*aa^*ab} \cdot \lambda = b^*aa^*ab^*$$

## SECTION 3.2 p. 91

# 10(b)  $M_0$ : $M_1$ : $M_2$ :

$$L(M) = \lambda \cdot ((a+bb).(ab)^*b)^*$$

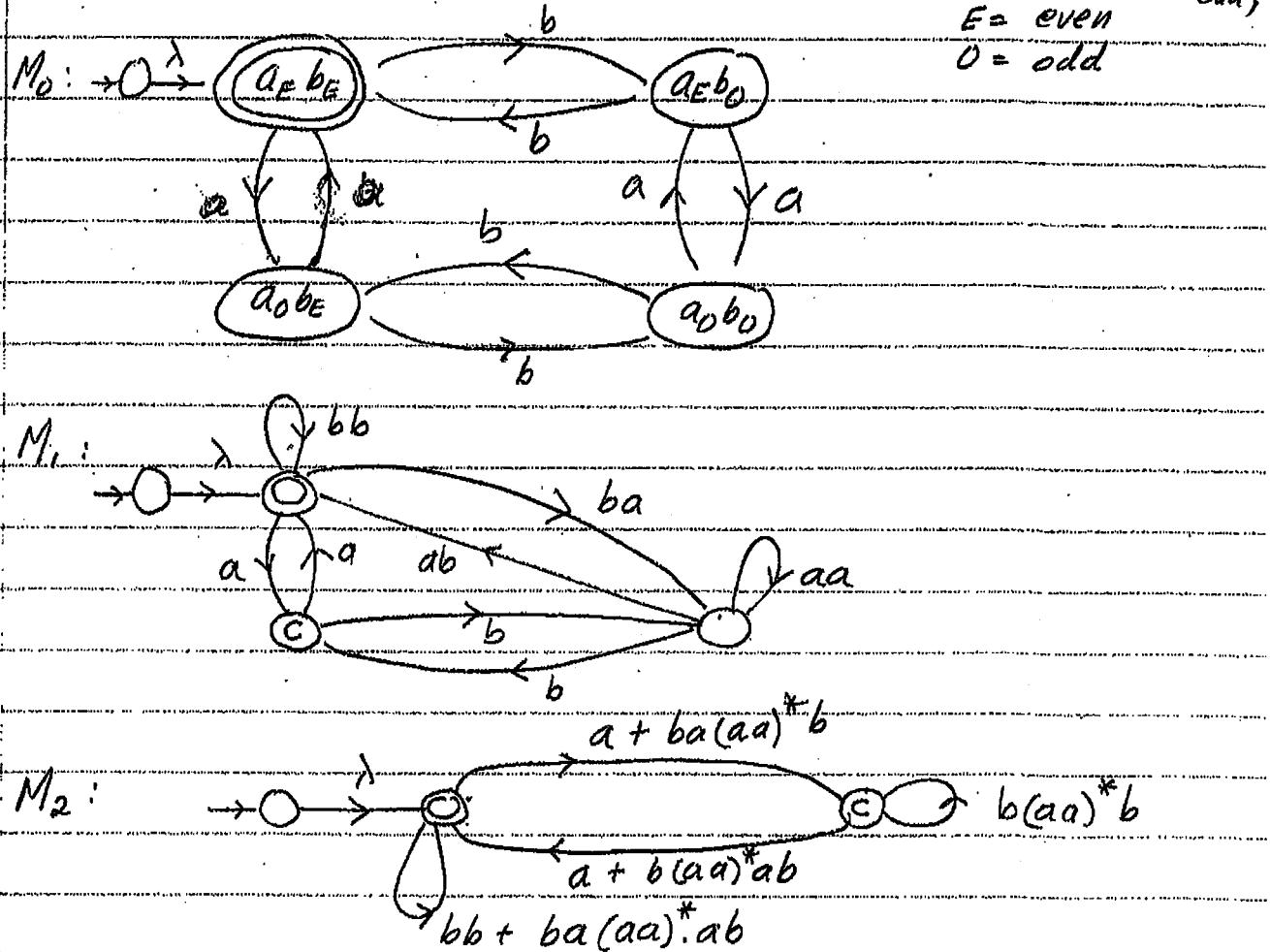
(c)  $M_0$ : $M_1$ : $M_2$ :

$$L(M) = a^* (b + (b+ba).(aa)^*\lambda.(a+\lambda))$$

(20)

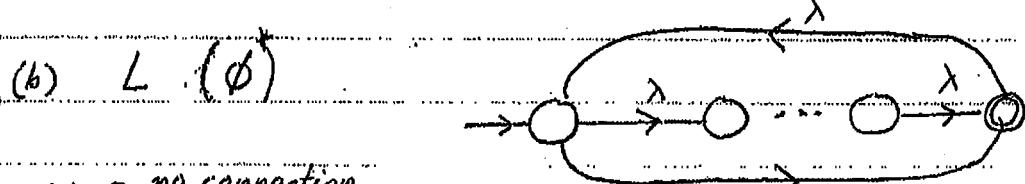
## SECTION 3.2 p. 91

- #15 (a) First find an nfa and then find the regular expression from your nfa.  $L = \{w : n_a(w) \& n_b(w) \text{ are both odd}\}$



$$L(M) = \lambda \cdot (bb + ba(aa)^*ab) \cdot ab + (a + ba(aa)^*b) \cdot (b(aa)^*b) \cdot (a + b(aa)^*ab)$$

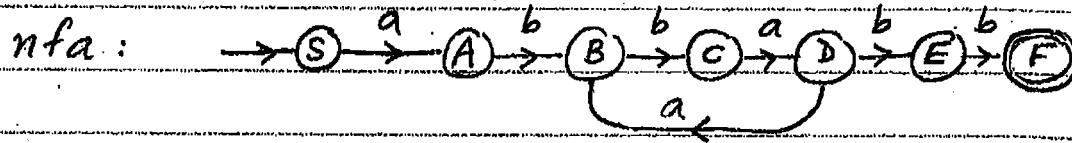
- #18 (a)  $L(a\emptyset)$ :  $\rightarrow O^\lambda \rightarrow O^\lambda \rightarrow O^\lambda \dots O^\lambda \rightarrow O$



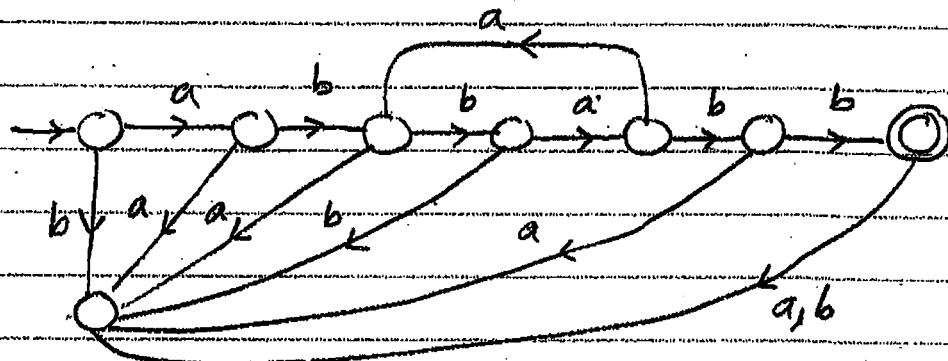
$\dots = \text{no connection.}$   
 $= \text{dead wire.}$

## SECTION 3.3 p. 99

#1. First find an nfa that accepts  $L(G)$  and then convert your nfa into a dfa.



dfa:



#3  $S \rightarrow aA, A \rightarrow aA, A \rightarrow B, B \rightarrow abB, B \rightarrow aB, B \rightarrow \lambda$ .

#4 The simplest thing to do is to find  $L(G)$  in Exercise 1 and then find a left-linear grammar for  $L(G)$ .

$$L(G) = abba. (aba)^* bb$$

Left-Lin. Grammar is  $S \rightarrow Abb, A \rightarrow Aabaabbba$

#5. (a) RLG:  $S \rightarrow aaA, A \rightarrow aA/B, B \rightarrow bB/bbb$

(b) LLG:  $S \rightarrow Bbbb, B \rightarrow Bb/A, A \rightarrow aa/Aa$

#6 RLG:  $S \rightarrow aaB, B \rightarrow bb, B \rightarrow ab, B \rightarrow abs$

$$S \rightarrow \lambda \quad L = (aab^*ab)^* \text{ here.}$$

Note: After you see the scheme, you will then realize that the 3rd production is not needed.

So a better ans. is:  $S \rightarrow aab/\lambda, B \rightarrow abs/bb,$

## SECTION 3.3 p. 99

- #7. Let  $L_i = \{\varphi \in \{a,b\}^*: \varphi \text{ has exactly } i \text{ a's}\}$   
 for  $i = 0, 1, 2 \& 3$ . Find regular grammars  $G_i$   
 for  $L_i$  and then find a grammar  $G$   
 which gives the union.

$G: S \rightarrow S_0/S_1/S_2/S_3, S_0 \rightarrow bS_0/\lambda,$   
 $S_1 \rightarrow bS_1/aA, A \rightarrow bA/\lambda,$   
 $S_2 \rightarrow bS_2/aB, B \rightarrow bB/aC, C \rightarrow bC/\lambda$   
 $S_3 \rightarrow bS_3/aD, D \rightarrow bD/aE, E \rightarrow bE/aF$   
 $F \rightarrow bF/\lambda$ .

- #11  $S \rightarrow Aab, A \rightarrow Ab, A \rightarrow aa, A \rightarrow Sa, S \rightarrow \lambda$   
 As in exercise 5 you don't really need the  
 3rd production. Here  $L = ?(\underline{aa}b^*ab)^*$ .

- #12 Let  $L_1 = \{a^n b^m : n \& m \text{ are even}\}$

$$L_2 = \{a^n b^m : n \& m \text{ are odd}\}$$

Then  $L = L_1 \cup L_2$ . Find Regular  
 grammars for  $L_1$  &  $L_2$  & then do the  
 union thing.

$$S_2 \rightarrow aaS_2/aB, B \rightarrow bbB/b$$

$$S \rightarrow S_1/S_2, S_1 \rightarrow aaS_1/A, A \rightarrow bbA/\lambda$$

- #13  $S \rightarrow aA/bB/\lambda, A \rightarrow aS/bC, B \rightarrow bS/aC, C \rightarrow bA/aB$ .

- #14 Hint: Find the corresponding nfa then convert.

(a)  $S \rightarrow \lambda/bB/aD, B \rightarrow bS/aC, C \rightarrow aB/bD$   
 $D \rightarrow aS/bC$

$$L = \{w \in \{a,b\}^* : n_a(w) + 3n_b(w) \text{ is even}\}$$