

SECTION 1.2 (ANSWERS OR HINTS) p. 28

#4 (a) $a b a a \cdot b a a \cdot a b \cdot a a, a a \cdot a a \cdot b a a \cdot a a, b a a \cdot a a \cdot a b \cdot a a \in L^*$
 $a b \cdot a a \cdot b a a \cdot a b \cdot a a$ is not in L^4 but the next two are in L^4

(b) $b a a \cdot a a a b \cdot a a \cdot a a \cdot b$ is not in L^* nor L^4 .

#5. $\bar{L} = \{\lambda, a, b, ab, ba\} \cup \{\varphi \in \{a, b\}^*: |\varphi| \geq 3\}$.

#25 (a) If $L = \{a^n b^{n+1} : n \geq 0\}$ then $L \neq L^*$ because
 $\lambda \notin L$ but $\lambda \in L^*$

(b) If $L = \{w : n_a(w) = n_b(w)\}$ then $L = L^*$. Prove this.

#6 Hint: $L \cup \bar{L} = V^*$ and V^* is infinite.

#7 No. No matter what L is, we will have
 $\lambda \in L^*$. So $\lambda \notin \bar{L}^*$. But $\lambda \in (\bar{L})^*$. So
we can never have $\bar{L}^* = (\bar{L})^*$.

#10 (a) TRUE (b) TRUE

#11 (a) $S \rightarrow bS, S \rightarrow Sb, S \rightarrow a$

(b) $S \rightarrow bS, S \rightarrow Sb, S \rightarrow a, S \rightarrow SS$

(c) $S \rightarrow AAA, A \rightarrow \lambda$

$A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$

(d) $S \rightarrow AAA, A \rightarrow AA$

$A \rightarrow bA, A \rightarrow Ab, A \rightarrow a$

#12 $L(G) = \{(ab)^n : n \geq 0\}$

#13. $L(G) = \emptyset$

#25 (b) $L = \{w : n_a(w) = n_b(w)\}$. Since $L^* = \bigcup_{k=0}^{\infty} L^k$, $L \subseteq L^*$.

Now suppose $\varphi \in L^*$. Then $\varphi = \varphi_1 \varphi_2 \cdots \varphi_k$ where
each $\varphi_i \in L$. So $n_a(\varphi) = n_a(\varphi_1) + n_a(\varphi_2) + \cdots + n_a(\varphi_k)$
 $= n_b(\varphi_1) + n_b(\varphi_2) + \cdots + n_b(\varphi_k) = n_b(\varphi)$ b.c. each $\varphi_i \in L$.
So $\varphi \in L$ b.c. $n_a(\varphi) = n_b(\varphi)$. So $L \subseteq L$. Hence $L^* = L$.

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(2)

- 14 (a) $S \rightarrow aSb/Sb/b$
 (b) $S \rightarrow aSbb/\lambda$
 (c) $S \rightarrow aaA, A \rightarrow aAb/ab$
 (d) $S \rightarrow aaaA, A \rightarrow aAb/\lambda$
 (e) $S \rightarrow AB, A \rightarrow aAb/Ab/b, B \rightarrow aBbb/\lambda$
 (f) $S \rightarrow A/B, A \rightarrow aAb/Ab/b, B \rightarrow aBbb/\lambda$
 (g) $S \rightarrow AAA, A \rightarrow aAb/Ab/b$
 (h) $S \rightarrow SA/\lambda, A \rightarrow aAb/Ab/b$

- 15 (a) $S \rightarrow SS/aaa/\lambda$
 (b) $S \rightarrow Saaa/aa/a$
 (c) $S \rightarrow Sa^6/a^5/a^4/a^3/a^2$
 (d) $S \rightarrow Sa^6/a^5/a^4/a^2/a/\lambda$

16. $S \rightarrow aSa/bSb/aa/bb$

17. $L(G) = \{\varphi a \varphi^T : \varphi \in \{a,b\}^*\}$ where $\varphi^T = \varphi$ with all a's replaced by b's and all b's replaced by a's.

18. (a) $S \rightarrow aA/AS, A \rightarrow AA/aAb/bAa/\lambda$
 (b) $S \rightarrow aS/AS/aA, A \rightarrow AA/aAb/bAa/\lambda$
 (c) $S \rightarrow abSa/aaSb/bSaa/SS/\lambda$

21. No. If $G_1 := S \rightarrow aSb/ab/\lambda$ and
 $G_2 := S \rightarrow aAb/ab, A \rightarrow aAb/\lambda$ then $\lambda \in L(G_1)$
 but $\lambda \notin L(G_2)$. Actually $L(G_1) = \{a^n b^n : n \geq 0\}$
 and $L(G_2) = \{a^n b^n : n \geq 1\}$

22. The only new production is $S \rightarrow SSS$, but we can simulate this from Ex. 1.12 by $S \rightarrow \underline{SS} \Rightarrow \underline{SSS}$.

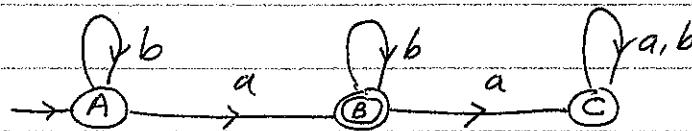
23. Hint: aa can be generated by first grammar but not by second one.

(3)

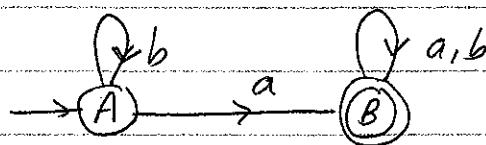
SECTION 2.1 p.47

- #1 0001 and 01001 will be accepted
 0000110 will not be accepted

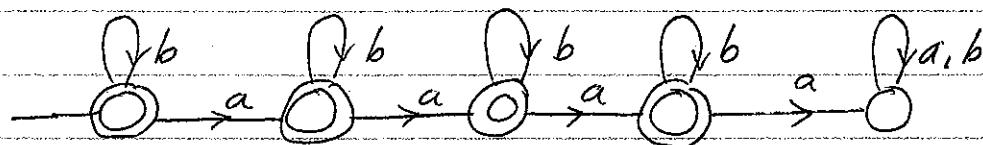
#2 (a)



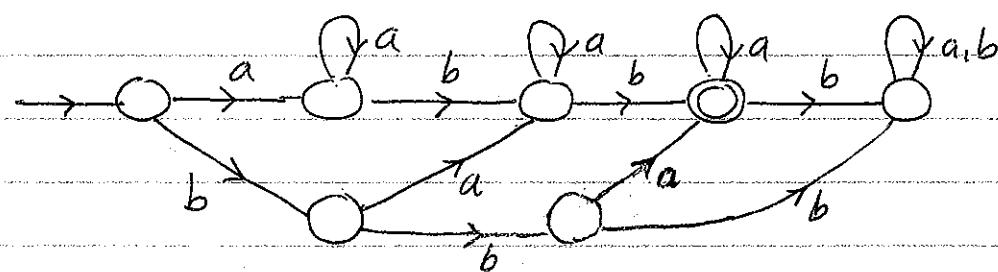
(b)



(c)



(d)



- #3. Let $\varphi \in \bar{L}$. Then φ will be rejected by M . So when we input φ in M we will end up at q_0 , q_1 , or q_2 . Since these are accepting states in M^c , φ will be accepted by M^c . So $\varphi \in L(M^c)$.
- Now let $\varphi \in L(M^c)$. Then φ will be accepted by M^c . So when we enter φ in M^c , we will end up at q_0 , q_1 , or q_2 . But these are rejecting states in M . So φ will be rejected by M . So $\varphi \notin L(M) = L$. So $\varphi \in \bar{L}$. Hence $\bar{L} = L(M^c)$.

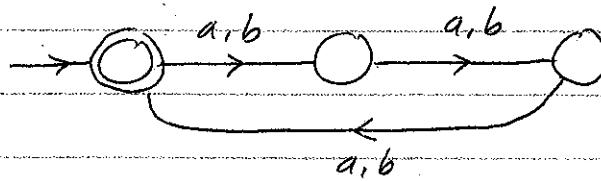
(4)

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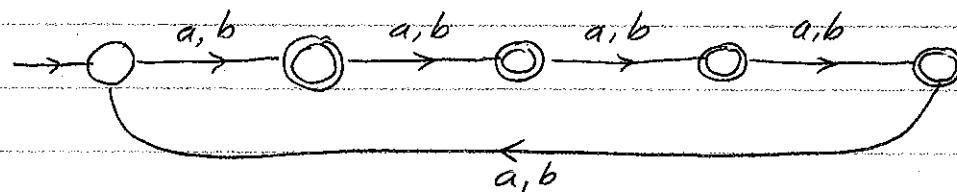
#4. Let $\varphi \in L(M)$. Then $\delta^*(q_0, \varphi) \in F$. Now

$$\begin{aligned}\varphi \in L(M) &\Leftrightarrow \varphi \notin L(M) \\ &\Leftrightarrow \delta^*(q_0, \varphi) \notin F \\ &\Leftrightarrow \delta^*(q_0, \varphi) \in Q - F \\ &\Leftrightarrow \varphi \in L(\hat{M})\end{aligned}$$

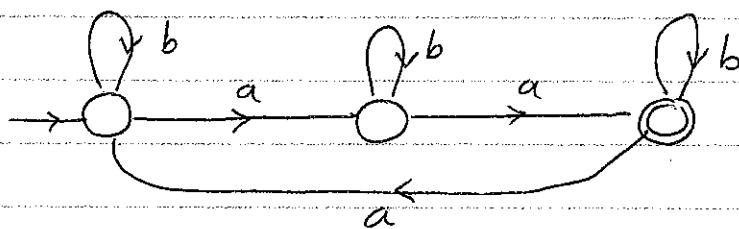
#7 (a)



(b)



(c)



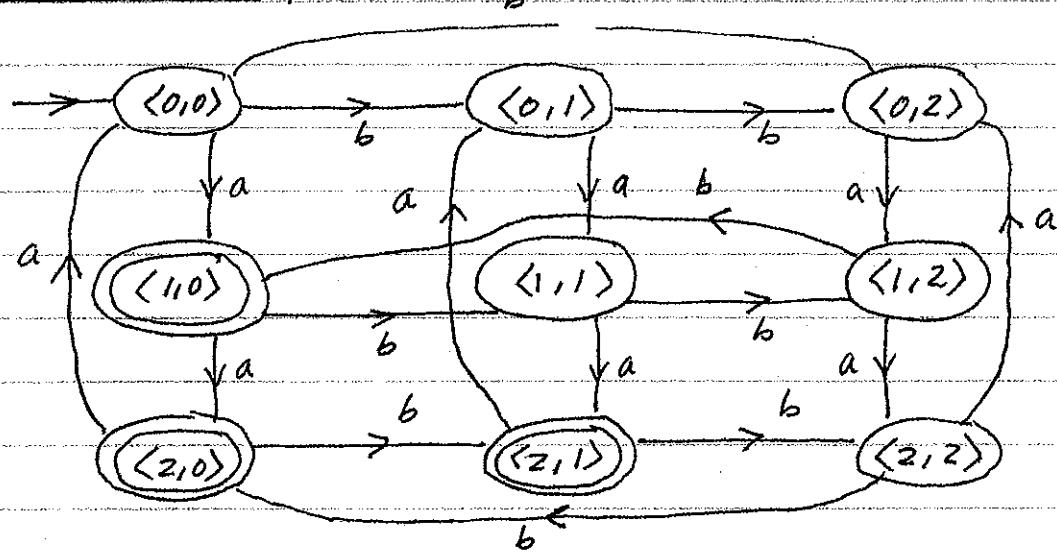
(d) Use the 9 states $\{(0,0), (0,1), (0,2), (1,0), \dots, (2,2)\}$
 $= \{0,1,2\} \times \{0,1,2\}$ to keep track of $n_a(w)$ & $n_b(w)$. The first component will be $n_a(w) \pmod{3}$ and the second will be $n_b(w) \pmod{3}$.

$\langle 0,0 \rangle$ will be the starting state b.c. $n_a(\lambda) = 0 = n_b(\lambda)$.
 $\langle 1,0 \rangle, \langle 2,0 \rangle$ & $\langle 2,1 \rangle$ will be accepting states b.c.
 $n_a(w) > n_b(w) \pmod{3}$ for these states.

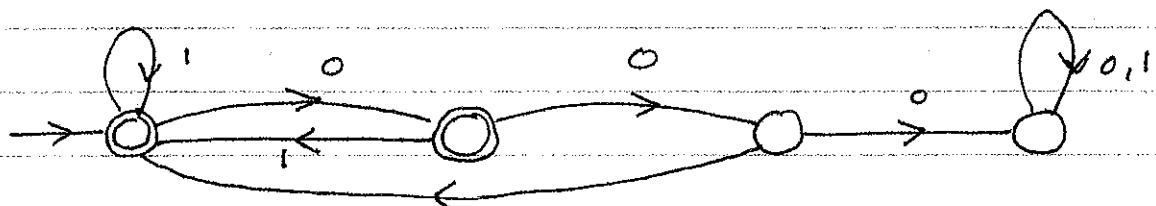
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(5)

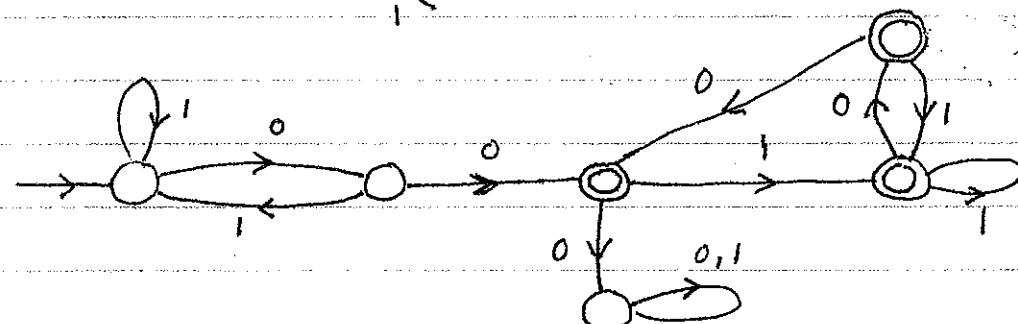
7(d)



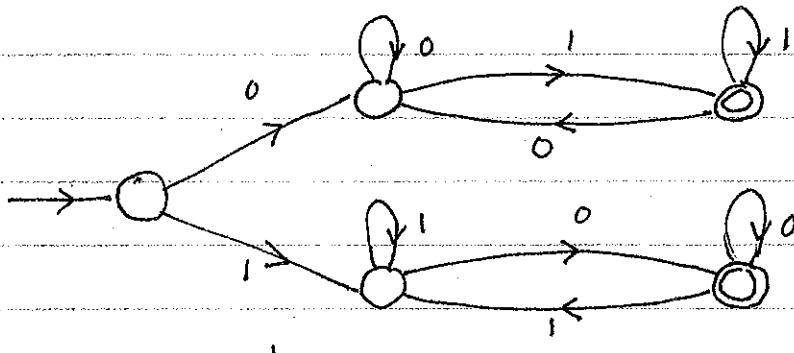
9(a)



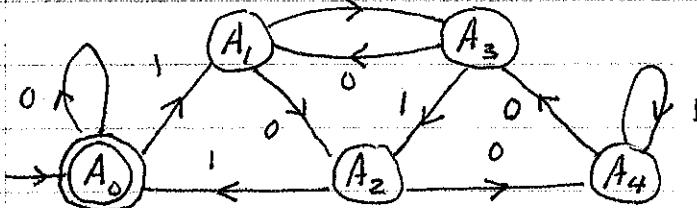
(b)



(c)



10.



Let A_i keep track of the binary value of $\varphi \pmod{5}$ where $\varphi = \text{input string so far}$. Initially $\varphi = \lambda$.

(6)

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#10 Example:

$$\begin{array}{ccccccc} 1 & 1 & 1 & 0 & 0 & 1 \\ A_0 & A_1 & A_3 & A_2 & A_4 & A_3 & A_2 \end{array}$$

Successive inputs: $(\lambda)_2 = 0 \rightarrow A_0$

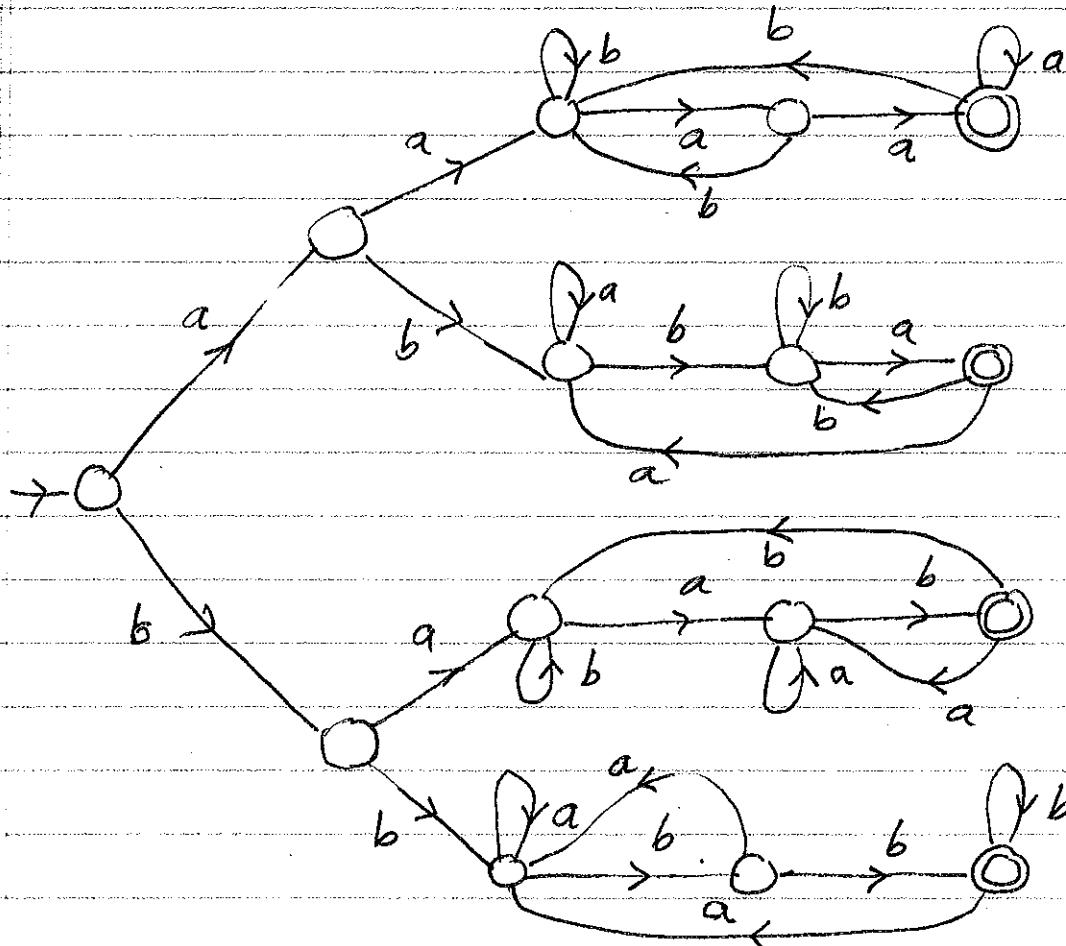
$$(\lambda)_2 = 1 \rightarrow A_1 \quad (\lambda)_2 = 3 \rightarrow A_3 \quad 7 = (111)_2 \rightarrow A_2$$

$$1110 = 14 \rightarrow A_4 \quad (11100)_2 = 28 \rightarrow A_3 \quad 111001 = 57 \rightarrow A_2$$

$$\text{Note: } (\varphi 0)_2 = 2(\varphi)_2 + 0$$

$$(\varphi 1)_2 = 2(\varphi)_2 + 1$$

#11



#12



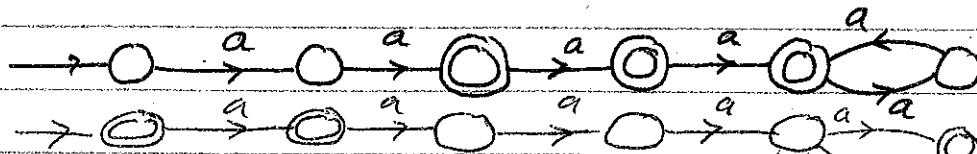
#13



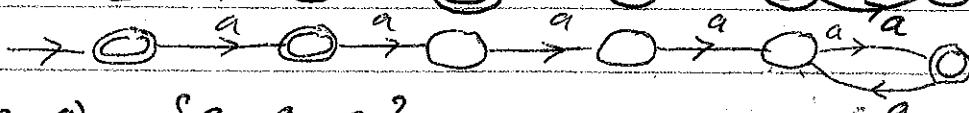
7

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#2



#3



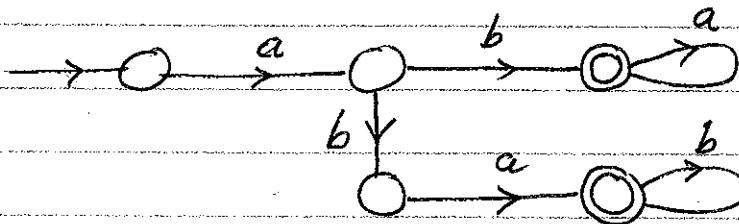
$$\#4 \quad \delta^*(q_0, a) = \{q_0, q_1, q_2\}$$

$$\delta^*(q_1, a) = \{q_0, q_1, q_2\}$$

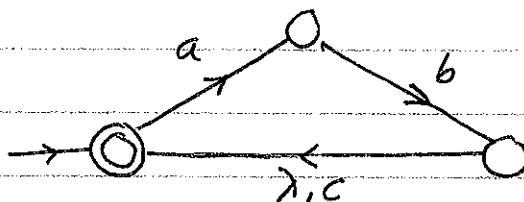
$$\#5 \quad \delta^*(q_0, 1010) = \{q_0, q_2\}$$

$$\delta^*(q_1, 00) = \emptyset$$

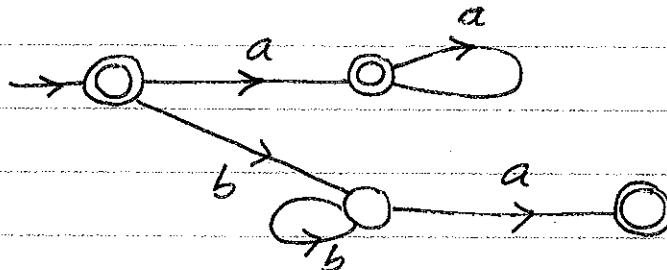
#7



#8

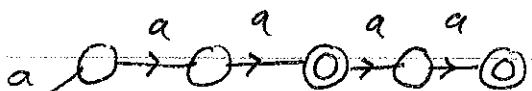


#11

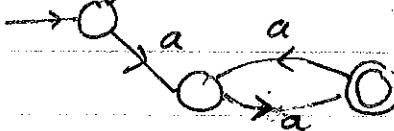
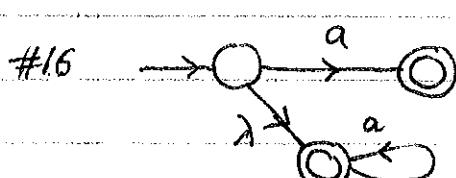


#12 01001, 000

#13



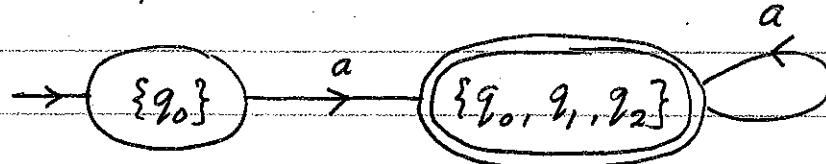
#16



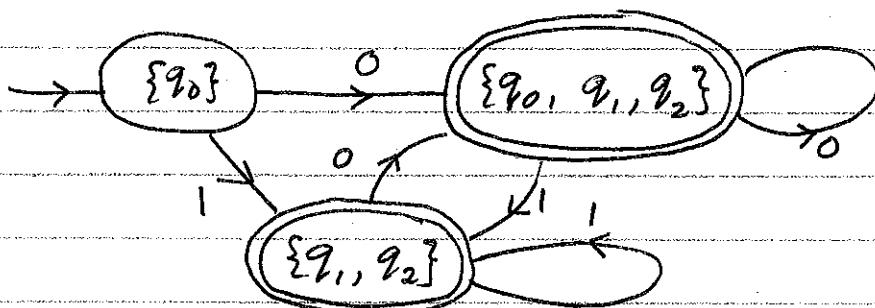
(8)

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#1

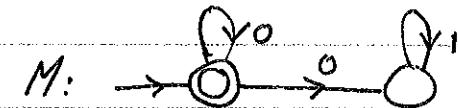


#3



#5 Yes; Hint: $L(M) = \{q \in \Sigma^*: \delta^*(q_0, q) \cap F \neq \emptyset\}$
 $\overline{L(M)} = \{q \in \Sigma^*: \delta^*(q_0, q) \cap F = \emptyset\}$

#6 No. The complement of $L(M)$ can include strings which are rejected by M , not because they drive M into a non-accepting state, but because M lacks transitions to process them.



$$\Sigma = \{0, 1\}, \quad 1 \notin L(M)$$

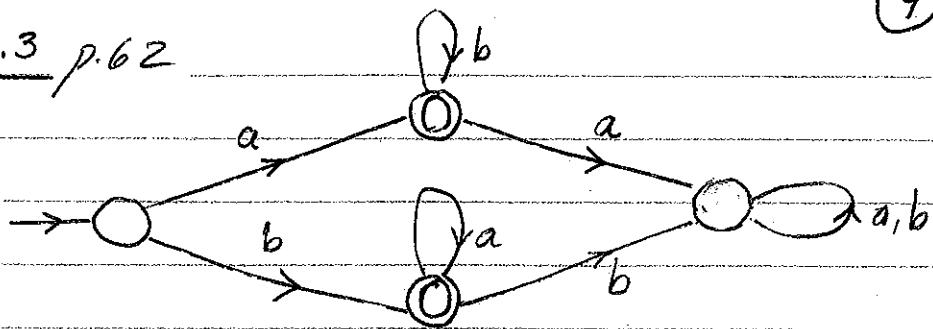
So $1 \in \overline{L(M)}$ but $1 \notin \{w \in \Sigma^*: \delta^*(q_0, w) \cap Q - F \neq \emptyset\}$
because $\delta^*(q_0, 1) = \emptyset$.

#7 (a) Hint: Let $M = \langle Q, \Sigma, \delta, q_0, F \rangle$. Change M to M' by adding λ -transitions from all accepting states in M to a new accepting state in M' and make all the accepting states in M , non-accepting in M' . Then check that $L(M') = L(M)$.

(9)

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#7(b) No.



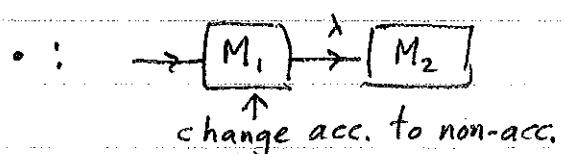
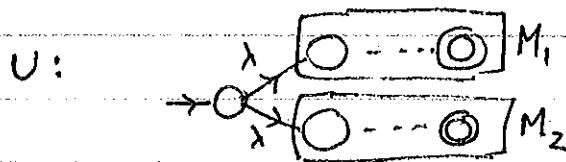
#10 Yes. First introduce a new initial state q_0' and add λ -transitions from it to all the initial states of M to get a new machine M' . In M' , q_0' will be the only initial state. Now M' will be an nfa, so we can convert it into a dfa M'' . The dfa M'' will have only one initial state and will be equivalent to our original M which had multiple initial states.

#11(a) One way is to just show that every finite language can be described by a reg. expression.

$$\text{Ex. } \{ab, baa, abab\} = (ab + baa + abab)$$

(b) We can also show that we can find an nfa which accepts any finite language by starting with simple nfa's & using union & concatenation.

$$\emptyset : \rightarrow \textcircled{O} \xleftarrow{\lambda} \textcircled{O} \quad \{b\} : \rightarrow \textcircled{O} \xrightarrow{\lambda} \textcircled{O} \quad \{ab\} : \rightarrow \textcircled{O} \xrightarrow{a} \textcircled{O}$$



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non-accepting

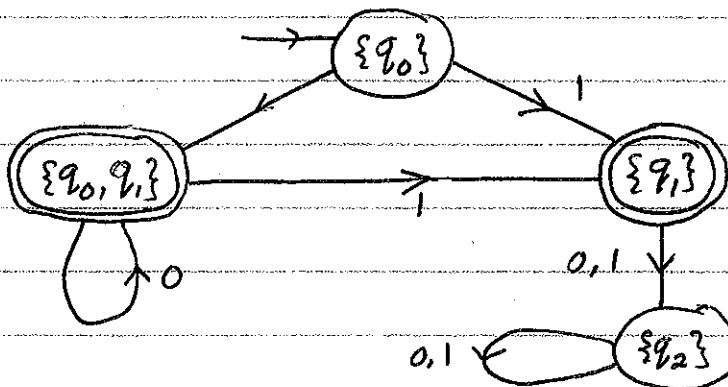
accepting states

$$\#1 \quad P_0 : \{\{q_0\}, \{q_2\}, \emptyset\}, \quad \{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$$

$$P_1 : \{\{q_0\}\}, \{\{q_2\}, \emptyset\}, \{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}\}, \{\{q_0, q_1\}, \{q_0, q_1, q_2\}\}$$

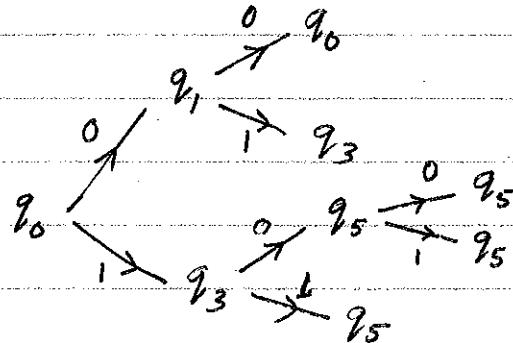
$$P_2 : = P_1$$

Reduced dfa:



#4.

(a)

 q_2 & q_4 are
inaccessible

$$(b) \quad P_0 : \{q_0, q_1\} \quad \{q_3, q_5\} \quad M^R : \xrightarrow{q_0} q_1 \xrightarrow{q_3} q_3$$

$$P_1 : \{q_0, q_1\} \quad \{q_3, q_5\} = P_0$$

#6. The conjecture is true. Suppose not. Then we can find a minimal DFA M for L such that \hat{M} is not minimal for \bar{L} . Now minimize \hat{M} to get a smaller DFA N for \bar{L} . By switching accepting & non-acc. states in N , we will get a DFA \hat{N} for $\bar{L} = L$, contradicting the minimality of M . So result is true.