SECTION 3.1 p. 75

#1. b, ab, ba, bb, aab, aba, abb, baa, bab, bba, bbb

#2. Yes.

#3 (a) see page 12 for detailed solution

(b) $(1 + 01)^*(0 + 1^*).\lambda^*$

(c) $(1 + 01)^*(0 + \lambda).\lambda$

#4 $aaaa^*.(bb)^*$

#5 $(aa)^*(bb)^* + a(aa)^*b(bb)^*$

#6 (a) $aaaaa^*.(\lambda + b + bb + bbb)$

(b) $(\lambda + a + aa + aaa).(\lambda + b + bb + bbb)$

#7 (a) $\{\lambda\}$ (b) ϕ

#8 Set of all strings that consists of a "b" surrounded by an even no. of a's on both sides or an odd number of a's on both sides.

$\{a^m b a^n : m - n \equiv 0 \pmod 2\}$

#9 $(ba + a)^*.b.(b + a)^*$

#10 We split $L = \{a^n b^m : n.m \geq 3,\ n \geq 1\ \&\ m \geq 1\}$ into three pieces according to the conditions

$n \geq 3,\ m = 1$ to get $\{a^n b : n \geq 3\}$

$n \geq 2,\ m = 2$ " $\{a^n bb : n \geq 2\}$

$n \geq 1,\ m \geq 3$ " $\{a^n b^m : n \geq 1,\ m \geq 3\}$

Then answer is $aaaa^*b + aaa^*bb + aa^*bbbb^*$

SECTION 3.1 (Number 3 redone) p.76

#3 (a) Let $R_1 = (\underline{1}+\underline{0}\underline{1})^* \cdot (\underline{0}+\underline{1}^*)$ and

$R_2 = (\underline{1}+\underline{0}\underline{1})^* \cdot (\underline{0}+\underline{\lambda})$

be the expression from Example 3.6.

Clearly $L(R_2) \subseteq L(R_1)$ because $\underline{0}+\underline{\lambda} \subseteq \underline{0}+\underline{1}^*$ $(\lambda \in \underline{1}^*)$.

Now let $\varphi \in L(R_1)$. Then

$\varphi = \alpha . \beta$ where $\alpha \in L((\underline{1}+\underline{0}\underline{1})^*)$ and $\beta = 0$ or $\beta \in L(\underline{1}^*)$

But if $\beta = 0$, then $\varphi \in L(R_2) = L((\underline{1}+\underline{0}\underline{1})^* . (\underline{0}+\underline{\lambda}))$
And if $\beta \in L(\underline{1}^*)$, then $\beta = 1^n$ for some $n \geq 0$. So

$\varphi = \alpha . 1^n$ with $\alpha \in L((\underline{1}+\underline{0}\underline{1})^*)$

$=$ 1's & (01)'s followed by n 1's

$\in L\ (\underline{1}+\underline{0}\underline{1})^* = L((\underline{1}+\underline{0}\underline{1})^* . \underline{\lambda})$

$\subseteq L\ (\underline{1}+\underline{0}\underline{1})^* . (\underline{0}+\underline{\lambda})$

$\therefore L(R_1) \subseteq L(R_2)$. Hence $L(R_1) = L(R_2)$

(b) $(\underline{0}\underline{1}+\underline{1})^* \cdot (\underline{\lambda}+\underline{0})$ and $(\underline{1}+\underline{0}\underline{1})^* \cdot (\underline{\lambda}+\underline{0}+\underline{1})$ are two other expressions which are equivalent to $(\underline{1}+\underline{0}\underline{1})(\underline{0}+\underline{1}^*)$.

\# 12 $L = a\,bbbb^*(a+b)(a+b)^*$

L = set of all strings which consists of an even no. of a's followed by an odd no. of b's

$= \{a^{2n} b^{2m+1} : n \geq 0, m \geq 0\}$

L^c = set of all strings of the form $a^n b^m$ with n odd or m even, or of strings with a "b" in front of an "a"

$= \{a^{2n+1} b^m : n \geq 0, m \geq 0\} \cup \{a^n b^{2m} : n \geq 0, m \geq 0\}$
$\cup$ {anything . ba . anything}

An expression for L^c is now easily seen to be

$$a(aa)^* b^* + a^*(bb)^* + (a+b)^* . ba . (a+b)^*$$

\# 13. $aa(a+b)^*aa + ab(a+b)^*ab + ba(a+b)^*ba + bb(a+b)^*bb$.

\# 14. A silly question. The answer is $(a+b)^*$.

\# 15. $(1 + 01)^* . 00 . (1+10)^*$

\# 16 (a) $(b+c)^* a (b+c)^*$

(b) Look at the four cases: no a's, one a, two a's and three a's. With these cases we get:

$(b+c)^* + (b+c)^* . a . (b+c)^* + (b+c)^* . a . (b+c)^* . a . (b+c)^*$
$+ (b+c)^* . a . (b+c)^* . a . (b+c)^* . a . (b+c)^*$

(c) Look at six cases: ...a...b...c..., ...a...c...b..., ...b...a...c..., ...b...c...a..., ...c...a...b..., ...c...b...a... .
Now insert $(a+b+c)^*$ for the dots.

\# 17. (a) $(\underline{0}+\underline{1})^* \cdot \underline{01}$ (f) $(\underline{0}+\lambda)(\underline{1}+\underline{00}+\underline{000})^*(\underline{0}+\lambda)$

(b) $(\lambda+\underline{0}+\underline{1}) + (\underline{0}+\underline{1})^* \cdot (\underline{00}+\underline{10}+\underline{11})$

(c) $(\underline{1}^*\underline{0}\,\underline{1}^*\underline{0}\,\underline{1}^*)^* + \underline{1}^*$ is one answer

$(\underline{1} + \underline{01}^*\underline{0})^*$ is another answer

(d) $(\underline{0}+\underline{1})^*(\underline{000} + \underline{00}(\underline{0}+\underline{1})^* \cdot \underline{00})(\underline{0}+\underline{1})^*$

(e) $(\underline{01}+\underline{1})^*(\lambda+\underline{0}+\underline{00} + \underline{000} + \underline{00} \cdot (\underline{10}+\underline{1})^* \underline{100})(\underline{10}+\underline{1})^*$

\# 20 (a) Clearly $L(r_1^*) \subseteq L((r_1^*)^*)$. Now let $\varphi \in L((r_1^*)^*)$. Then

φ = a string of strings from r_1^*

= a string of strings of strings from r_1

= a string of strings from r_1

$\in L(r_1^*)$

$\therefore \quad L((r_1^*)^*) \subseteq L(r_1^*)$

Thus $L((r_1^*)^*) = L(r_1^*)$ i.e. $(r_1^*)^* \equiv r_1^*$

(b) Again clearly $L((r_1+r_2)^*) \subseteq L(r_1^*(r_1+r_2)^*)$ because $\lambda \in L(r_1^*)$.

Now let $\varphi \in L(r_1^*(r_1+r_2)^*)$. Then

$\varphi = \alpha \cdot \beta$ with $\alpha \in L(r_1^*)$ & $\beta \in L((r_1+r_2)^*)$

$= \alpha_1 \alpha_2 \cdots \alpha_m \cdot \beta_1 \beta_2 \cdots \beta_n$ with the α_i's in $L(r_1)$ and β_j's in $L(r_1+r_2)$

$= \alpha_1 \alpha_2 \cdots \alpha_m \beta_1 \cdots \beta_n$ with the α_i's and β_j's in $L(r_1+r_2)$

$\in L((r_1+r_2)^*)$

$\therefore \quad L(r_1^*(r_1+r_2)^*) \subseteq L((r_1+r_2)^*)$.

So $L(r_1^*(r_1+r_2)^*) = L((r_1+r_2)^*)$ i.e. $r_1^*(r_1+r_2)^* \equiv (r_1+r_2)^*$.

#20 (c) First observe that $L(r_1^* r_2^*) \supseteq L(r_1 + r_2)$ bec. $\lambda \in L(r_1^*)$ and $\lambda \in L(r_2^*)$. So $L((r_1^* r_2^*)^*) \supseteq L((r_1+r_2)^*)$

Now let $\varphi \in L((r_1^* r_2^*)^*)$. Then

$\varphi =$ a string of things which are made of a string from r_1^* followed by a string from r_2^*

$=$ a string of things which are made up of strings of things in r_1 followed by strings of things in r_2.

$=$ a string of things from either r_1 or r_2

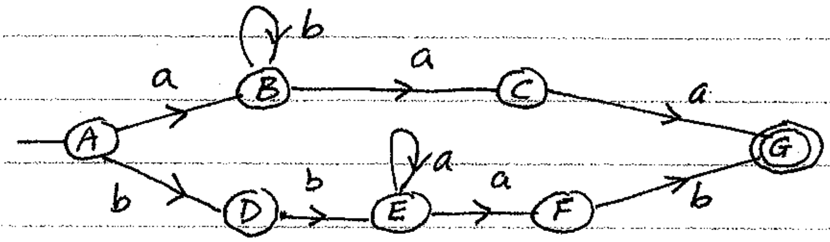
$\in L((r_1+r_2)^*)$.

So $L((r_1^* r_2^*)^*) \subseteq L((r_1+r_2)^*)$. Thus $L((r_1^* r_2^*)^*) = L((r_1+r_2)^*)$ i.e. $(r_1+r_2)^* \equiv (r_1^* r_2^*)^*$

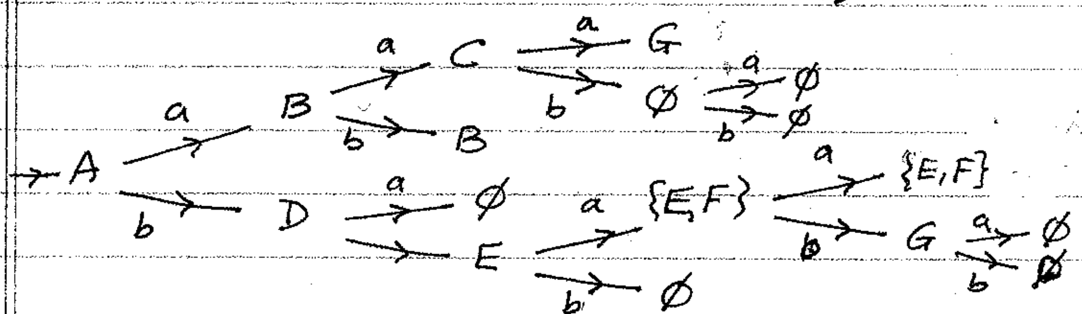
d) false. $(\underline{a}.\underline{b})^* \neq \underline{a}^*.\underline{b}^*$ because $abab \in (\underline{a}.\underline{b})^*$ but $abab \notin \underline{a}^*.\underline{b}^*$.

SECTION 3.2 p. 87

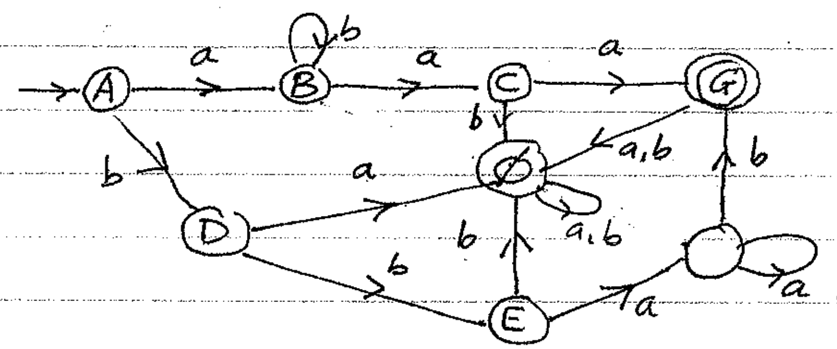
#1 $M =$



#2. (a) First convert M into a dfa M_D

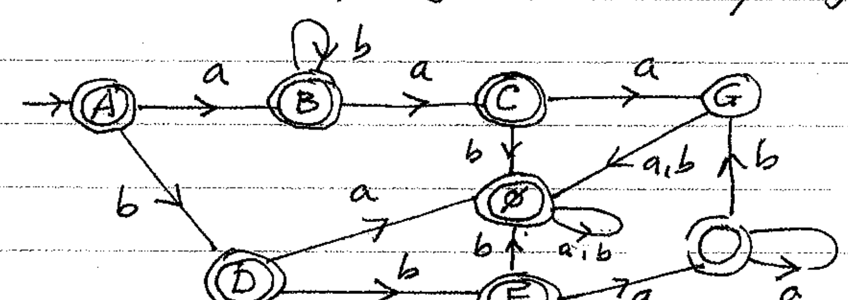


$M_D =$

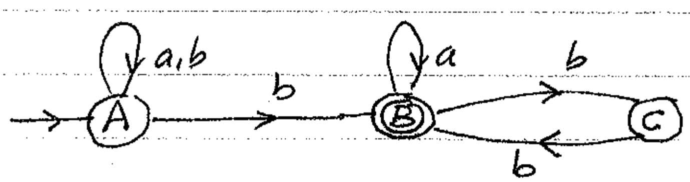


(b) Then switch accepting & non-accepting states

$M_D^c =$



#3.

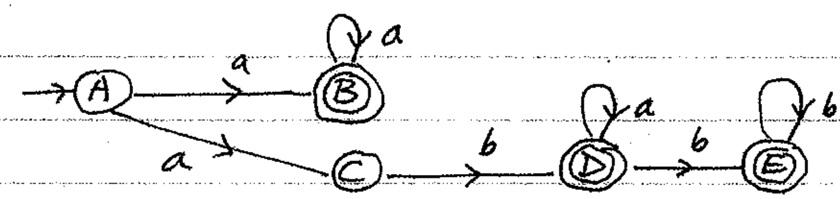


Section 3.2 p.87

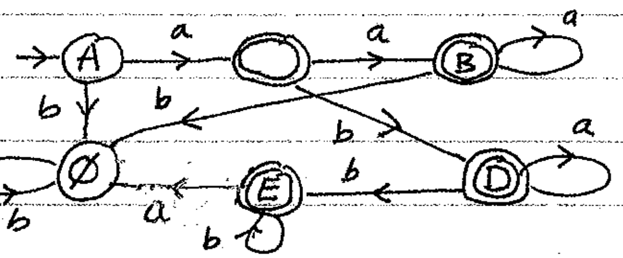
#4. First find an nfa and then convert it into a dfa

(a) M =

nfa :



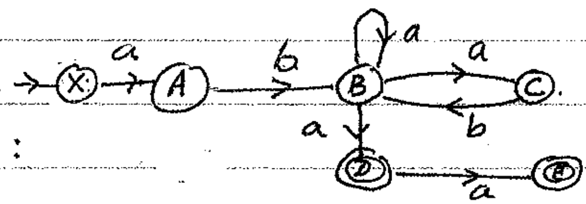
M_D =



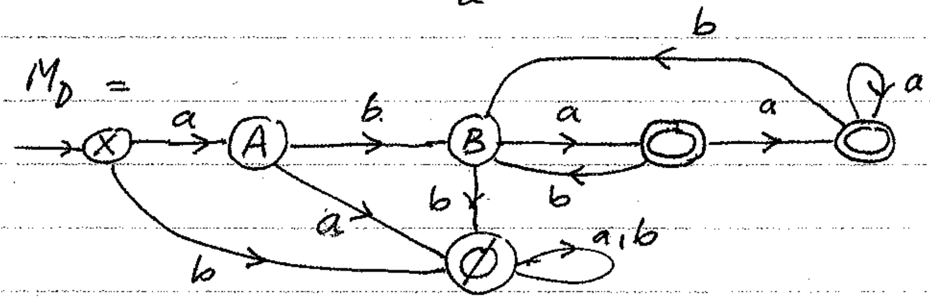
= dfa

(b) M =

nfa:

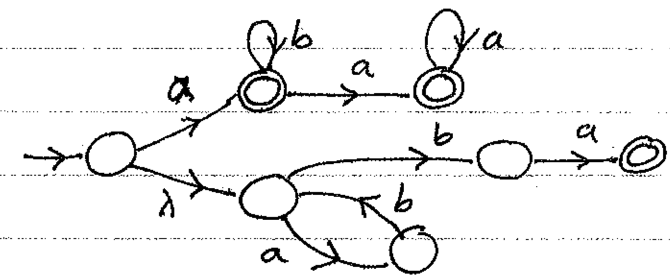


dfa: M_D =



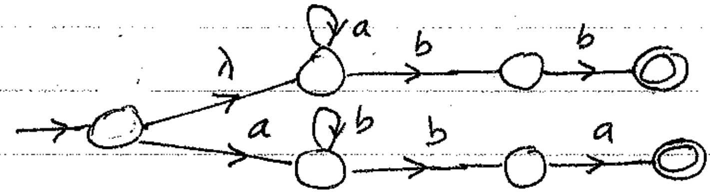
#5. (a) nfa,

M =



Now convert M into a dfa M_D.

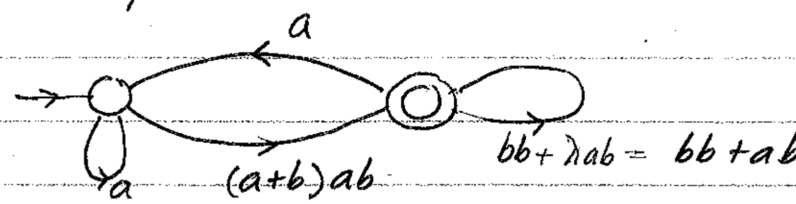
#7 nfa M =



Now convert M into a dfa M_D and then minimize M_D by using the Partition Algorithm.

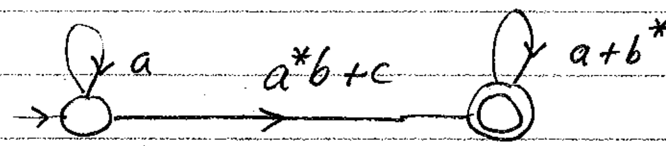
SECTION 3.2 p. 88

#8 (a)



(b) $$\underbrace{a^*}_{r_1^*} \cdot \underbrace{(a+b)ab}_{r_2} \cdot \Big(\underbrace{(bb+ab)}_{r_4} + \underbrace{a}_{r_3} \cdot \underbrace{a^*(a+b)ab}_{r_1^* \cdot r_2} \Big)^*$$

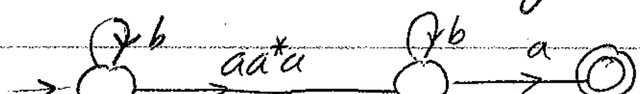
#9



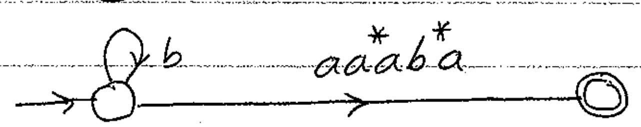
$$L(M) = \underbrace{a^*}_{r_1^*} \cdot \underbrace{(a^*b+c)}_{r_2} \cdot \underbrace{(a+b^*)}_{r_4}{}^*$$

#10 (a) Since this is an easy example, you can instantly see that $L(M) = b^*aa^*ab^*a$. But let's see how the algorithm proceeds

M_1



M_2



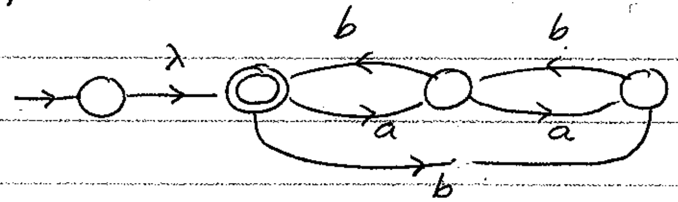
$$L(M) = \underbrace{b^*}_{r_1^*} \cdot \underbrace{aa^*ab^*a}_{r_2} \cdot \underbrace{(\phi)^*}_{r_4 + r_3 r_1^* r_2} \quad \leftarrow r_1^* r_2 \big(\underbrace{r_4}_{\phi} + \underbrace{r_3 r_1^* r_2}_{=\phi}\big)^*$$

bec. $r_3 = \phi$

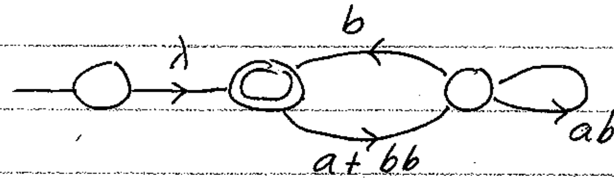
$$= b^*aa^*ab^*a \cdot \lambda = b^*aa^*ab^*a.$$

SECTION 3.2 p. 88

\# 10(b) M_0 :



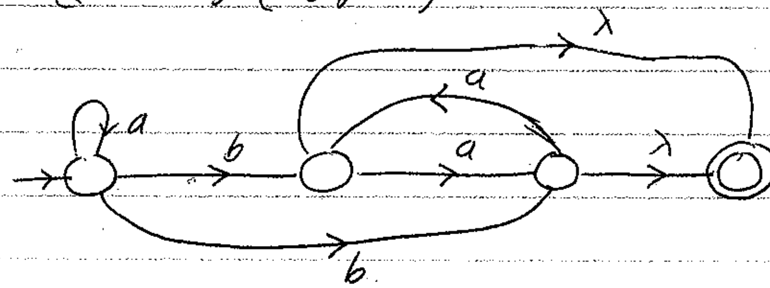
M_1 :



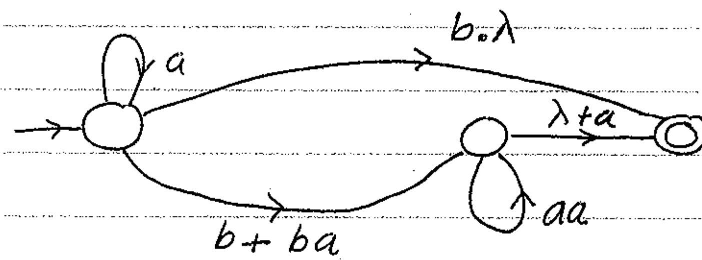
M_2 :

$L(M) = \lambda . ((a+bb).(ab)^*. b)^*$

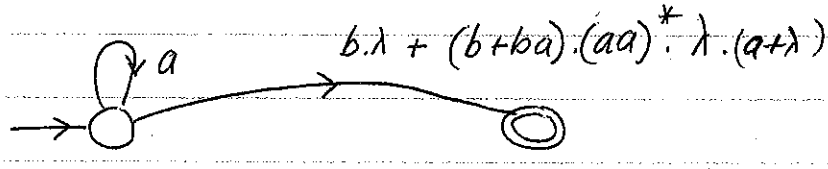
(c) M_0 :



M_1 :



M_2 :



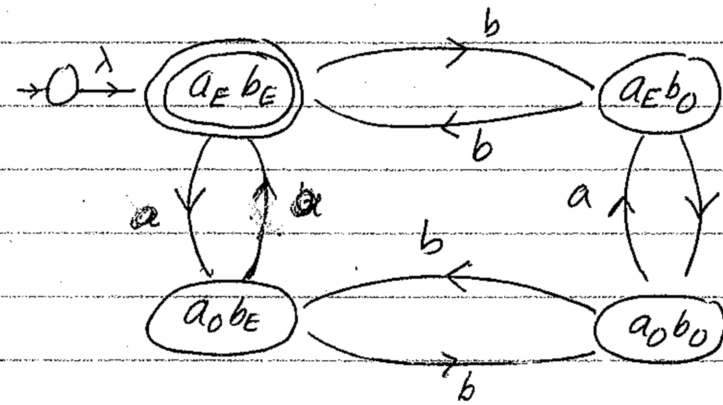
$L(M) = a^*. \left(b + (b+ba).(aa)^*. \lambda . (a+\lambda)\right)$

SECTION 3.2 p. 88

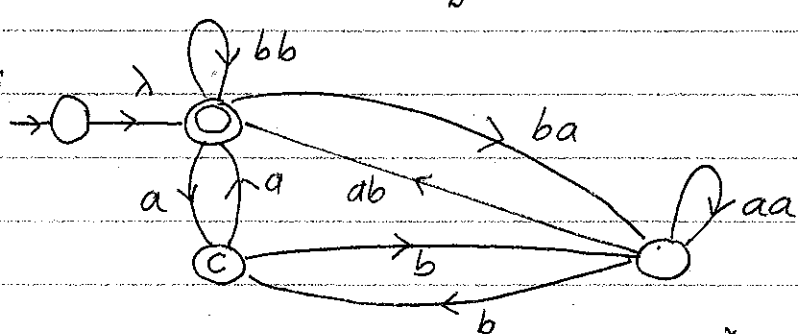
#13 (a) First find an nfa and then find the regular expression from your nfa.

E = even
O = odd

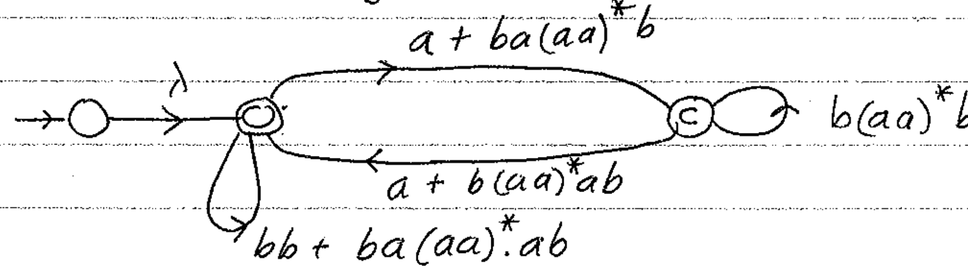
M_0:



M_1:

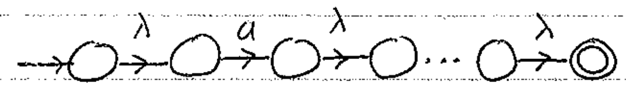


M_2:

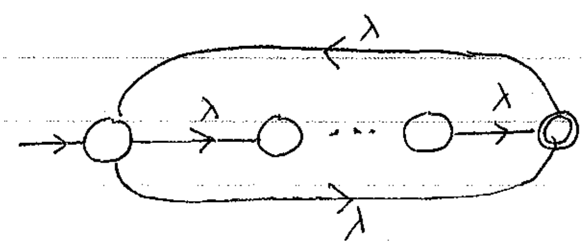


$$L(M) = \lambda.\left(bb + ba.(aa)^*.ab + (a+ba(aa)^*b).(b(aa)^*b)^*.(a+b(aa)^*.ab)\right)^*$$

#18 (a) $L(a\emptyset)$:



(b) $L(\emptyset^*)$

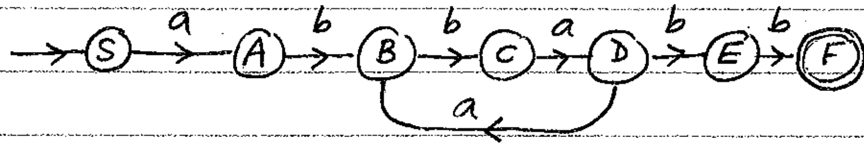


.... = no connection.
= dead wire

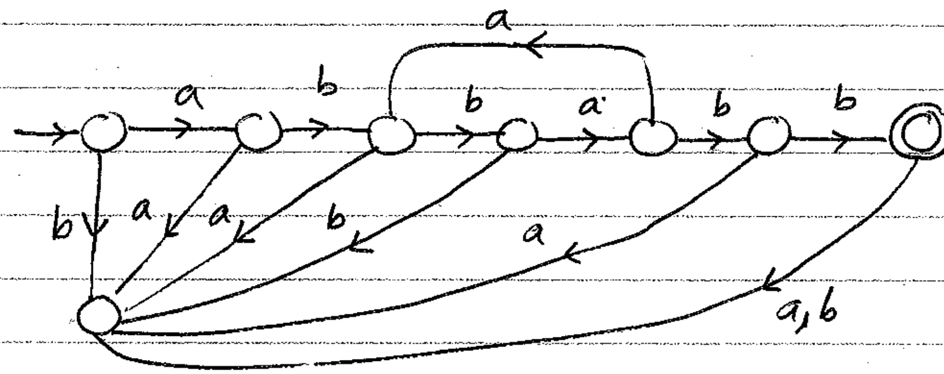
SECTION 3.3 p. 96

#1. First find an nfa that accepts $L(G)$ and then convert your nfa into a dfa.

nfa :



dfa :



#2. $S \to aA$, $A \to aA$, $A \to B$, $B \to abB$, $B \to aB$, $B \to \lambda$.

#3. The simplest thing to do is to find $L(G)$ in Exercise 1 and then find a left-linear grammar for $L(G)$.

$L(G) = \underline{abba}.(\underline{aba})^* \underline{bb}$

Left-Lin. Grammar is $S \to Abb$, $A \to Aaba \mid abba$

#4. (a) RLG: $S \to aaA$, $A \to aA \mid B$, $B \to bB \mid bbb$

(b) LLG: $S \to Bbbb$, $B \to Bb \mid A$, $A \to aa \mid Aa$

#6. RLG: $S \to aaB$, $B \to bB$, $B \to ab$, $B \to abS$

$S \to \lambda$.

Note: After you see the scheme, you will then realize that the 3rd production is not needed. So a better ans. is: $S \to aaB \mid \lambda$, $B \to abS \mid bB$.

SECTION 3.3 p. 96

\#7. Let $L_i = \{\varphi \in \{a,b\}^* : \varphi$ has exactly i a's$\}$ for $i = 0, 1, 2$ & 3. Find regular grammars G_i for L_i and then find a grammar G which gives the union.

G: $S \to S_0 / S_1 / S_2 / S_3$, $\quad S_0 \to bS_0 / \lambda$,
$S_1 \to bS_1 / aA$, $\quad A \to bA / \lambda$,
$S_2 \to bS_2 / aB$, $\quad B \to bB / aC$, $\quad C \to bC / \lambda$
$S_3 \to bS_3 / aD$, $\quad D \to bD / aE$, $\quad E \to bE / aF$
$F \to bF / \lambda$.

\#10. $S \to Aab$, $A \to Ab$, $A \to aa$, $A \to Saa$, $S \to \lambda$
As in exercise 5 you don't really need the 3rd production.

\#11 Let $L_1 = \{a^n b^m : n$ & m are even$\}$
$L_2 = \{a^n b^m : n$ & m are odd$\}$
Then $L = L_1 \cup L_2$. Find Regular grammars for L_1 & L_2 & then do the union thing: $\quad S_2 \to aaS_2 / aB$, $\quad B \to bbB / b$
$S \to S_1 / S_2 \quad S_1 \to aaS_1 / A$, $\quad A \to bbA / \lambda$

\#12 $S \to aA / bB / \lambda$, $\quad A \to aS / bC$, $\quad B \to bS / aC$, $\quad C \to bA / aB$.

\#13 Hint: Find the corresponding nfa then convert.
(a) $S \to \lambda / bB / aD$, $\quad B \to bS / aC$, $\quad C \to aB / bD$
$D \to aS / bC$