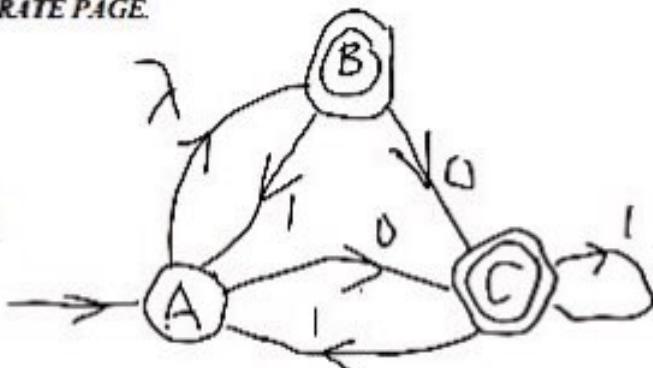


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit.
BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1.(a) Define what is a *regular expression over the alphabet* $V = \{b, c, d\}$.
 (b) Let M be the NFA on the right. Find a DFA, M_D which is *equivalent* to M .



- (15) 2. Find *regular expressions*, E_1 and E_2 , which describe the languages, L_1 and L_2 , below.
 (a) $L_1 = \{\varphi \in \{0,1\}^*\mid \varphi \text{ contains both } 110 \text{ and } 01 \text{ as substrings}\}$.
 Indicate how 10100110 is described by your E_1 by putting dots between characters.
 (b) $L_2 = \{\varphi \in \{b,c\}^*\mid \varphi \text{ contains at most one occurrence of the string } cc\}$.
 Indicate how $c b c c b b c$ is described by your E_2 by putting dots between characters.

- (20) 3. (a) Define what it means for two states p & q to be *indistinguishable* in a DFA, M .
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent *reduced machine*, M_R .

	\rightarrow [A]	B	C	[D]	[E]	F	[G]
0	F	B	F	C	G	C	E
1	B	G	D	B	B	A	F

- (15) 4. (a) Let $f(\varphi) = [1 + 2.n_b(\varphi) - 3.n_c(\varphi)] \pmod{4}$. Find a DFA, M which accepts the language, $L_4 = \{ \varphi \in \{b,c\}^* \mid f(\varphi) \text{ is } 2 \text{ or } 3 \pmod{4} \}$.
 (b) If $\varphi = cbcbc$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

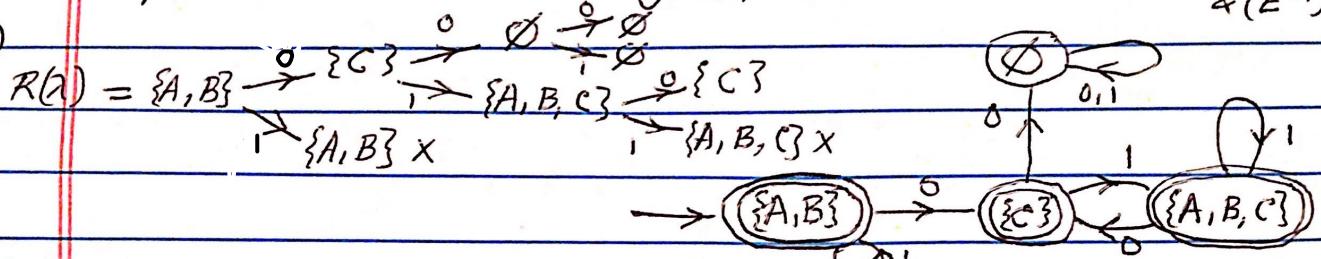
- (20) 5. (a) Find a *context-free grammar* G which generates the language
 $L_5 = \{a^k b^n \mid n \geq 2k+3, k \geq 0\} \cup \{b^k c^n \mid 0 \leq n \leq 3k+4, k \geq 0\}$.
 (b) Find *derivations* from your G for each of the strings: (i) $a^2 b^8$ and (ii) $b^4 c^4$.

- (15) 6. Let A , B , and C be languages based on the *alphabet* $\{a,b\}$.
 (a) Is it always true that $(A \cdot C) - (B \cdot C) \subseteq (A - B) \cdot C$?
 (b) Is it always true that $(A \cdot C) \cap (B \cdot C) \subseteq (A \cap B) \cdot C$? (Justify your answers.)

MAD 3512 - Theory of Algorithms Florida Int'l Univ.
 Solutions to Test #1 Fall 2026

1. (a) A regular expression over the alphabet $\{b, c, d\}$ is defined recursively as follows. (i) b, c, d, λ and \emptyset are regular expressions. If $E & F$ are reg. expr., then so are $(E+F)$, $(E \cdot F)$ & (E^*) .

(b)



2(a) ... 110 ... 01 ... , ... 01 ... 110 ... , ... 1101 ... , ... 0110 ...

$$E_1 = (0+1)^* \cdot [110(0+1)^*01 + 01 \cdot (0+1)^*110 + 1101 + 0110] \cdot (0+1)^*$$

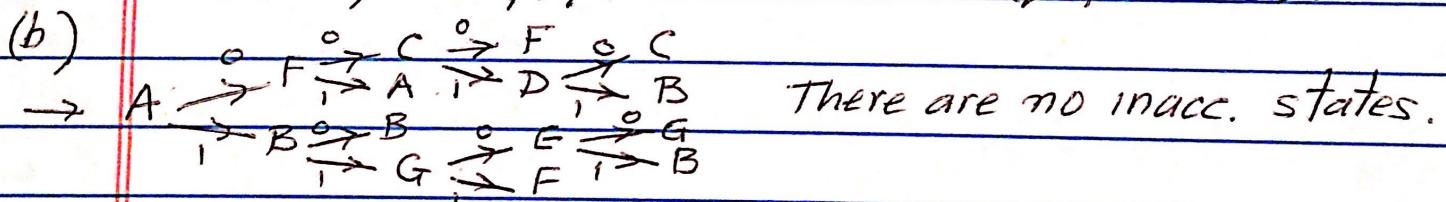
Two ways 1. 01. 00. 110. λ or 1010. 0110. λ .

(b) no cc & ends in b or is λ , no cc & ends in c, one cc

$$E_2 = (b+c\bar{b})^* + (b+c\bar{b})^* \cdot c + (b+c\bar{b})^* \cdot cc + (b+c\bar{b})^* \cdot (b+c\bar{b})^*.$$

One way cb. cc. b. bc

3(a) Two states $p & q$ in a DFA are indistinguishable if for each $\varphi \in T^*$, $s^*(p, \varphi) \in A(M) \Leftrightarrow s^*(q, \varphi) \in A(M)$.



$$P_0 : \{A, D, E, G\} \quad \{B, C, F\}$$

$$P_1 : \{A, D\} \{E, G\} \quad \{B, C, F\}$$

$$P_2 : \{A, D\} \{E, G\} \quad \{B\} \{C, F\}$$

$$P_3 : \{A, D\} \{E\} \{G\} \quad \{B\} \{C, F\}$$

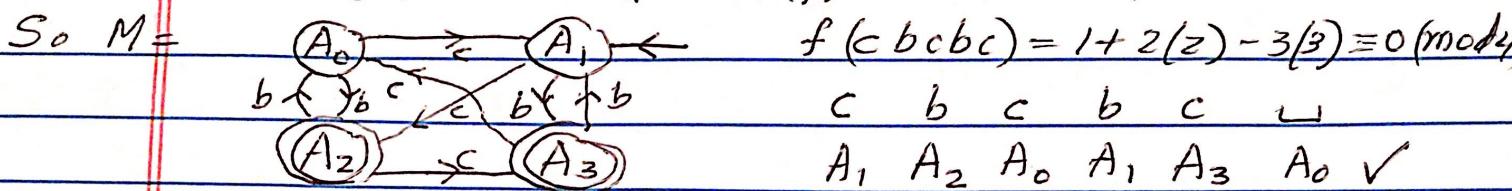
$$P_4 : \{A, D\} \{E\} \{G\} \quad \{B\} \{C, F\} = P_3$$

$$M_2 = \begin{array}{c|ccccc} & \{A, D\} & \{E\} & \{G\} & \{B\} & \{C, F\} \\ \hline 0 & \{C, F\} & \{G\} & \{E\} & \{B\} & \{C, F\} \\ 1 & \{B\} & \{B\} & \{C, F\} & \{G\} & \{A, D\} \end{array}$$

4(a) Let A_i ($i=0, 1, 2, 3$) keep track of the fact that the part of the string processed is $i \pmod{4}$. Then A_2 & A_3 will be the accepting states & A_0 will be the initial state because $f(\lambda) = 1 + 2n_b(\lambda) - 3n_c(\lambda) = 1 - 2(0) - 3(0) \equiv 1 \pmod{4}$.

$$f(\varphi b) = 1 + 2n_b(\varphi b) - 3n_c(\varphi b) = f(\varphi) + 2 \pmod{4}$$

$$f(\varphi c) = 1 + 2n_b(\varphi c) - 3n_c(\varphi c) = f(\varphi) - 3 \pmod{4} = f(\varphi) + 1 \pmod{4}$$



5(a) $S \rightarrow A/B$, $A \rightarrow aAb\bar{b} / Ab/b\bar{b}b$, $B \rightarrow bB\bar{D}\bar{D} / \bar{D}\bar{D}B\bar{D}\bar{D}$, $\bar{D} \rightarrow c/c$

$$(b) \rightarrow S \Rightarrow A \Rightarrow aAb\bar{b} \Rightarrow aaAb\bar{b}bb \Rightarrow a^2Ab\bar{b}a^4 \Rightarrow a^2b^3\bar{b}b^4 = a^2b^8$$

$$\rightarrow S \Rightarrow B \Rightarrow bB\bar{D}\bar{D} \Rightarrow b\bar{D}^4\bar{D}^3 \Rightarrow b\bar{c}^3\bar{D}^3 \Rightarrow bcc\bar{D}^2\bar{D}^3$$

$$\Rightarrow bccc\bar{D}^3 \Rightarrow bcccc\bar{D}^3 \Rightarrow bc^4\bar{a}^2\bar{D}^2 \Rightarrow bc^4\bar{a}^2\bar{D}^2 \Rightarrow bc^4\bar{a}^2$$

$$= bc^4$$

6(a) YES. Let $\varphi \in (A \cdot C) - (B \cdot C)$. Then $\varphi = \alpha \cdot \gamma$ with $\alpha \in A$, $\gamma \in C$ and $\alpha \cdot \gamma \notin B \cdot C$. Now if $\alpha \in B$, then $\alpha \cdot \gamma$ would be in $B \cdot C$.

Since $\alpha \cdot \gamma \notin BC$, $\alpha \notin B$. $\therefore \alpha \in (A - B)$ & $\gamma \in C$. $\therefore \alpha \cdot \gamma \in (A - B) \cdot C$

So if $\varphi \in (A \cdot C) - (B \cdot C)$, then $\varphi \in (A - B) \cdot C$. $\therefore (A \cdot C) - (B \cdot C) \subseteq (A - B) \cdot C$

(b) NO. Let $A = \{ab\}$, $B = \{\alpha\}$, and $C = \{\lambda, b\}$. Then

$$A \cdot C = \{ab, abb\} \text{ and } B \cdot C = \{\alpha, ab\}$$

$$\text{So } (A \cdot C) \cap (B \cdot C) = \{ab, abb\} \cap \{\alpha, ab\} = \{ab\}$$

$$\text{Also } A \cap B = \{ab\} \cap \{\alpha\} = \emptyset. \text{ So}$$

$$(A \cap B) \cdot C = \emptyset \cdot \{\lambda, b\} = \emptyset. \therefore (A \cdot C) \cap (B \cdot C) \not\subseteq (A \cap B) \cdot C$$

because $ab \in (A \cdot C) \cap (B \cdot C)$ but $ab \notin (A \cap B) \cdot C$

So $(A \cdot C) \cap (B \cdot C)$ is not always a subset of $(A \cap B) \cdot C$

END