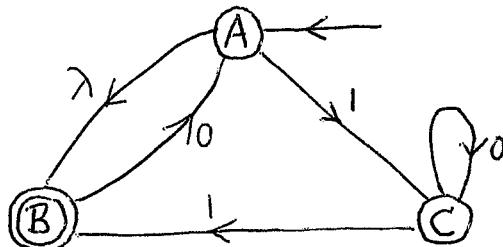


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit.
 BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1.(a) Define what is a *regular expression* over the alphabet $V = \{0, 1, 2\}$.
 (b) Let M be the NFA on the right. Find a DFA, M_c which recognizes $L(M)^c$.



- (15) 2. Find regular expressions, E_1 and E_2 , which describe the languages, L_1 and L_2 , below.
 (a) $L_1 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains both } 011 \text{ and } 101 \text{ as substrings}\}$.
 Indicate how 10100110 is described by your E_1 by putting dots between characters.
 (b) $L_2 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains at most one occurrence of the string } bb\}$.
 Indicate how $ababbaba$ is described by your E_2 by putting dots between characters.

- (20) 3. (a) Define what it means for two states p & q to be *indistinguishable* in a DFA, M .
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent reduced machine, M_R .

	→(A)	B	C	(D)	(E)	F	(G)
0	C	B	C	F	G	F	E
1	B	G	D	B	B	A	C

- (15) 4. (a) Let $f(\varphi) = [2.n_b(\varphi) - n_a(\varphi) - 3] \pmod{4}$. Find a DFA, M which accepts the language, $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is } 2 \text{ or } 3 \pmod{4}\}$.
 (b) If $\varphi = ababb$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

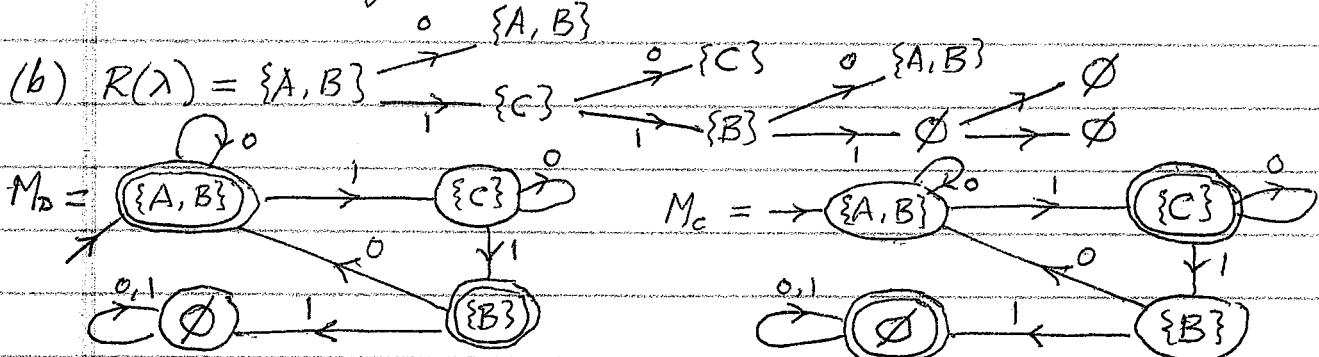
- (20) 5. (a) Find a *context-free grammar* G which generates the language
 $L_5 = \{a^k b^n: n \geq 3k + 2, k \geq 0\} \cup \{b^k c^n: 2 \leq n \leq 2k+3, k \geq 0\}$.
 (b) Find *derivations* from your G for each of the strings: (i) $a^1 b^6$ and (ii) $b^2 c^5$.

- (15) 6. Let A , B , and C be languages based on the alphabet $\{0,1\}$.
 (a) Is it always true that $(B.A) - (C.A) \subseteq (B - C).A$?
 (b) Is it always true that $(B.E) \cap (D.E) \subseteq (B \cap D).E$? (Justify your answers.)

1(a) A regular expression over $\{0, 1, 2\}$ is defined recursively as follows

(i) $0, 1, 2, \lambda$, and \emptyset are regular expressions.

(ii) If E & F are regular expressions, then so are $(E+F)$, $(E \cdot F)$, & (E^*) .



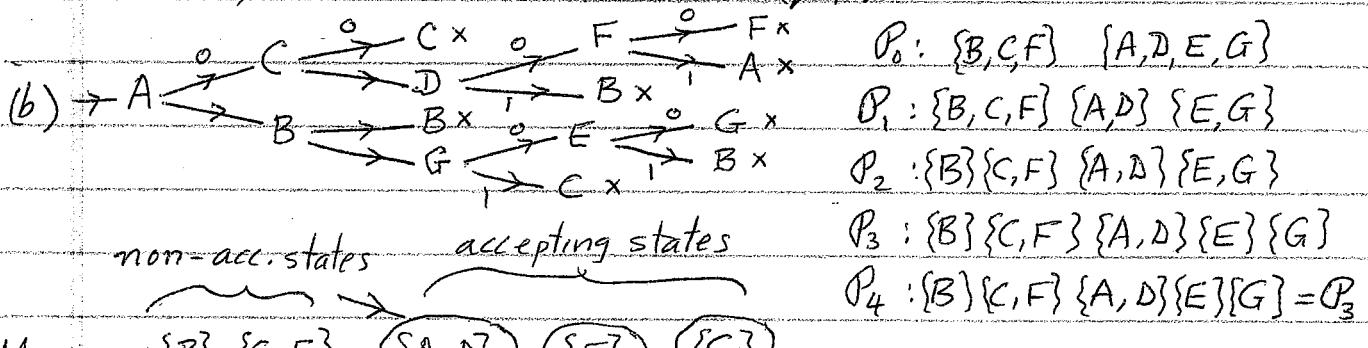
2(a) $E_1 = (\underline{0+1})^* \cdot \underline{101} \cdot (\underline{0+1})^*, \underline{011} \cdot (\underline{0+1})^* + (\underline{0+1})^* \cdot \underline{011} \cdot (\underline{0+1})^* \cdot \underline{101} \cdot (\underline{0+1})^*$

 $\quad \quad \quad + (\underline{0+1})^* \cdot \underline{101} \cdot \underline{0} \cdot \underline{011} \cdot \underline{0} + (\underline{0+1})^* \cdot \underline{101} \cdot (\underline{0+1})^* + (\underline{0+1})^* \cdot \underline{01101} \cdot (\underline{0+1})^*$

(b) $E_2 = (\underline{a+b}a)^* + (\underline{a+b}a) \cdot \underline{b} + (\underline{a+b}a)^* \cdot \underline{bb} \cdot (\underline{a+b}a)^*$

 $\quad \quad \quad a. \underline{ba}. \underline{bb}. \underline{ab}. a$

3(a) We say that p & q are indistinguishable in a DFA M , if for each $\varphi \in T^*$, $s^*(p, \varphi) \in A(M) \Leftrightarrow s^*(q, \varphi) \in A(M)$.



M_R	$\{B\} \{C, F\}$	$\{A, D\}$	$\{E\}$	$\{G\}$
0	$\{B\} \{C, F\}$	$\{C, F\}$	$\{G\}$	$\{E\}$
1	$\{G\} \{A, D\}$	$\{B\}$	$\{B\}$	$\{C, F\}$

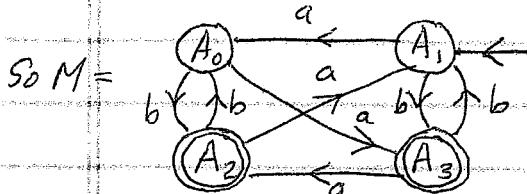
4(a) Let A_i ($i=0, 1, 2, 3$) keep track of the fact that the part of the string processed so far is $i \pmod{4}$. Then A_2 & A_3 will

be the accepting states & A_1 will be the initial state

$$\text{because } f(\lambda) = 2n_b(\lambda) - n_a(\lambda) - 3 = 2(0) - 0 - 3 \equiv 1 \pmod{4}.$$

$$\text{Now } f(\varphi a) = 2n_b(\varphi a) - n_a(\varphi a) - 3 = [2n_b(\varphi) - n_a(\varphi) - 3] - 1 \equiv f(\varphi) - 1 \pmod{4}$$

$$\text{and } f(\varphi b) = 2n_b(\varphi b) - n_a(\varphi b) - 3 = [2n_b(\varphi) - n_a(\varphi) - 3] + 2 \equiv f(\varphi) + 2 \pmod{4}$$



$$(b) f(\bar{a}ba\bar{b}b) = 2(3) - 2 - 3 \equiv 1 \pmod{4}$$

$$a \ b \ a \ b \ b \ \sqcup \\ A_1 \ A_0 \ A_2 \ A_1 \ A_3 \ A_1 \quad \text{Answer} = 1 \pmod{4}$$

$$5(a) S \rightarrow A/B, A \rightarrow aAbbb/D, D \rightarrow Db/bb, B \rightarrow bBEE/c^cE, E \rightarrow c/\lambda.$$

$$(b)(i) \rightarrow S \Rightarrow A \Rightarrow aAbbb \Rightarrow aDbbb \Rightarrow aDbbbb \Rightarrow abbbbbb = a'b^6$$

$$(ii) \rightarrow S \Rightarrow B \Rightarrow bBEE \Rightarrow bbBEEEE \Rightarrow bbccE EEEE \Rightarrow b^2c^2c EEEE \\ \Rightarrow b^2c^3c EEE \Rightarrow b^2c^4c EE \Rightarrow b^2c^5\lambda E \Rightarrow b^2c^5\lambda\lambda = b^2c^5.$$

6(a) YES. Let $\varphi \in (B.A) - (C.A)$. Then $\varphi \in B.A$ and $\varphi \notin C.A$.

So φ can be written as $\varphi = \beta.\alpha$ with $\beta \in B$ and $\alpha \in A$.

Now β cannot be in C , otherwise $\varphi = \beta.\alpha$ would be in $C.A$, which contradicts the fact that $\varphi \notin C.A$. So

$\beta \notin C$. Hence $\beta \in B-C$ and since $\alpha \in A$, we get

$$\varphi = \beta.\alpha \in (B-C).A. \text{ Thus } (B.A) - (C.A) \subseteq (B-C).A.$$

(b) NO. Let $B = \{1\}$ and $D = \{10\}$ and $E = \{\lambda, 0\}$. Then

$$(B.E) \cap (D.E) = (\{1\} \cdot \{\lambda, 0\}) \cap (\{10\} \cdot \{\lambda, 0\}) \\ = \{1, 10\} \cap \{10, 100\} = \{10\}.$$

$$\text{But } (B \cap D).E = (\{1\} \cap \{10\}).\{\lambda, 0\} = \emptyset \cdot \{\lambda, 0\} = \emptyset$$

$$\text{So } (B.E) \cap (D.E) \neq \emptyset = (B \cap D).E \text{ in this case.}$$

Hence $(B.E) \cap (D.E)$ will not always be a subset of $(B \cap D).E$.

END