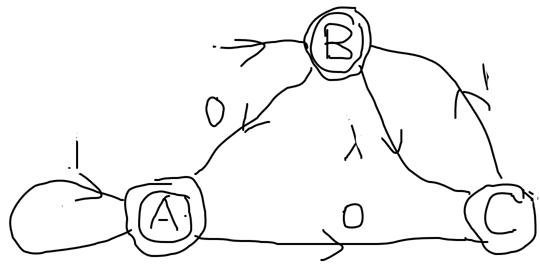
MAD 3512 - THEORY OF ALGORITHMS FLORIDA INT'L UNIV.
TEST #1 - SPRING 2021 TIME: 75 min.

Answer all 6 questions. ***No calculators, notes, or on-line data are allowed.*** *An unjustified answer will receive little or no credit.* ***Draw a line to separate each of the 6 solutions to the 6 questions.***

(15) 1.(a) Define what is the ***reaching set*** $R(\varphi)$ of a string φ in a *NFA, M.*
(b) Let *M* be the *NFA* on the right. Find a *DFA*, M_C which *accepts* $L(M)^C$.



(15) 2. Find *regular expressions*, E_1 and E_2, which describe the languages, L_1 and L_2, below.
(a) $L_1 = \{\varphi \in \{0,1\}^*: \ \varphi$ contains both *10* and *001* as substrings$\}$.
Indicate how 01011001 is described by your E_1 by putting dots between characters.
(b) $L_2 = \{\varphi \in \{b,c\}^*: \ \varphi$ contains *exactly two* occurrences of the string *bb*$\}$.
Indicate how bcbbcbcbb is described by your E_2 by putting dots between characters.

(20) 3. (a) Define what it means for a state *p* to be ***inaccessible*** in a *DFA, M.*
(b) Check for *inaccessible* states, then *partition* the states of the *DFA* below into *blocks of indistinguishable states,* and find the equivalent *reduced machine,* M_R.

	A	B	$\boxed{C}$	D	$\rightarrow \boxed{E}$	$\boxed{F}$	$\boxed{G}$
0	D	B	D	A	A	G	F
1	E	G	B	C	B	B	A

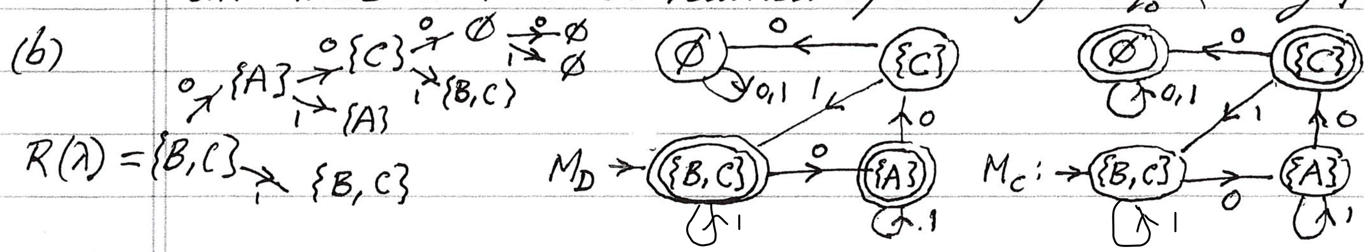
(15) 4. (a) Let $f(\varphi) = [2.n_c(\varphi) - 3n_b(\varphi) - 1]$ (mod 4). Find a *DFA, M* which accepts the language, $L_4 = \{ \varphi \in \{b,c\}^*: \ f(\varphi)$ *is 0 or 2* (mod 4) $\}$.
(b) If $\varphi = cbcbb$ find $f(\varphi)$ & check that it agrees with your *DFA* with φ as input.

(20) 5. (a) Find a *context-free grammar* G which generates the language
$L_5 = \{a^n b^k: \ n \geq 2k + 1 \ \ \& \ \ k \geq 0\} \cup \{b^n c^k: \ 0 \leq n \leq 3k+2 \ \ \& \ \ k \geq 0\}$.
(b) Find *derivations* from your G for each of the strings: (i) $a^6 b^2$ and (ii) $b^4 c^1$.

(15) 6. Let A, B, and C be languages based on the *alphabet* $\{0,1\}$.
(a) Is it always true that $(B.A) - (B.C) \subseteq B.(A - C)$?
(b) Is it always true that $(B.A) \cap (B.C) \subseteq B.(A \cap C)$? (***Justify your answers.***)

1 (a) The reaching set $R(\varphi)$ of a string φ in an NFA M is the set of all states that can be reached by starting at q_0 & using φ.

(b)



2(a) $E_1 = (0+1)^*(10.(0+1)^*.001 + 001.(0+1)^*.10 + 1001 + 0010)(0+1)^*$

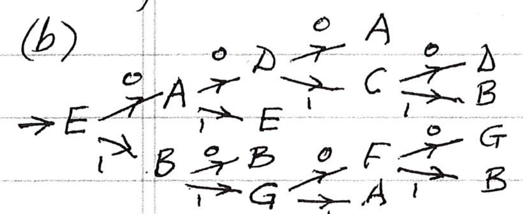
$0.\ 10.\ 11.\ 001.\ \lambda$

(b) $E_2 = (c+bc)^*.bbc.(c+bc)^*.bb.(c+cb)^* + (c+bc)^*.bbb.(c+cb)^*$

$bc.\ bbc.\ bc.\ bb.\ \lambda$

3(a) The state p is inaccessible in a DFA, M, if there is no string w such that $\delta^*(q_0, w) = p$.

(b)



$\mathcal{P}_0$: {A, B, D} {C, E, F, G}

$\mathcal{P}_1$: {A, B, D} {C, E} {F, G}

$\mathcal{P}_2$: {A, D} {B} {C, E} {F, G}

$\mathcal{P}_3$: {A, D} {B} {C, E} {F} {G}

$\mathcal{P}_4$: {A, D} {B} {C, E} {F} {G} = $\mathcal{P}_3$

	{A, D}	→ {B}	{C, E}	{F}	{G}
0	{A, D}	{B}	{A, D}	{G}	{F}
1	{C, E}	{G}	{B}	{B}	{A, D}

4(a) Let A_i $(i = 0,1,2,3)$ keep track of the fact that the part of the string processed is $i \pmod 4$. Then A_0 & A_2 will be the accepting states & A_3 will be the initial state because $f(\varphi)$

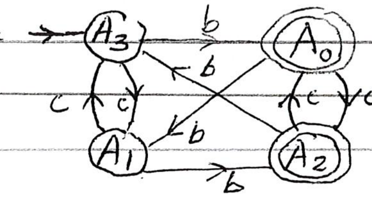
$$f(\lambda) = 2n_c(\lambda) - 3n_b(\lambda) - 1 = 0 - 0 - 1 \equiv 3 \pmod 4.$$

Also $f(\varphi b) = 2n_c(\varphi b) - 3n_b(\varphi b) - 1 = 2n_c(\varphi) - 3n_b(\varphi) - 1 - 3 \equiv f(\varphi) + 1.$

$f(\varphi c) = 2n_c(\varphi c) - 3n_b(\varphi c) - 1 = \underbrace{2n_c(\varphi) - 3n_b(\varphi) - 1}_{f(\varphi)} + 2 \equiv f(\varphi) + 2. \pmod 4$

$= 2[n_c(\varphi) + 1]$ $= -3[n_b(\varphi) + 1]$

$-3 \equiv 1 \pmod 4$

4(b) So $M =$ 

Check: $c \ b \ c \ b \ b \ \sqcup$
$A_2 \ A_1 \ A_2 \ A_0 \ A_1 \ A_2$ (p. 2)

$f(cbcbb) = 2(2) - 3(3) - 1 = -6 \equiv 2 \pmod 4$

5(a) $\to S$, $S \to A/C$, $A \to aaAb/D$, $D \to aD/a$

$C \to BBBCc\,/\,BB$, $B \to b/\lambda$.

(b)(i) $\to S \Rightarrow A \Rightarrow aaAb \Rightarrow aaaaAbb \Rightarrow a^4Db^2 \Rightarrow a^4aDb^2 \Rightarrow \underbrace{a^4aab^2}_{= a^6b^2}$.

(ii) $\to S \Rightarrow C \Rightarrow BBBCc \Rightarrow BBBBBc \Rightarrow \lambda BBBBc \Rightarrow \lambda bBBBc$

$\Rightarrow \lambda bbBBc \Rightarrow \lambda bbbBc \Rightarrow \lambda bbbbc = b^4c^1$.

6(a) YES. Let $\varphi \in (B.A) - (B.C)$. Then $\varphi = \beta.\alpha$ with $\beta \in B$ & $\alpha \in A$ and with $\beta.\alpha \notin B.C$. Now if α was in C, then $\beta.\alpha$ would be in $B.C$ which would contradict $\beta.\alpha \notin B.C$. So α cannot be in C. $\therefore$ $\alpha \in A$ & $\alpha \notin C$. $\therefore$ $\alpha \in A - C$. Thus $\varphi = \beta.\alpha \in B.(A-C)$ because $\beta \in B$ & $\alpha \in A - C$. Hence $(B.A) - (B.C) \subseteq B.(A-C)$ is always true.

(b) NO. Let $B = \{0, 01\}$, $A = \{1\}$, and $C = \{11\}$. Then

$B.\{A \cap C\} = \{0,01\}.\emptyset = \emptyset$ because A has one element and C has one element and these two elements are different.

Also $B.A = \{0,01\}.\{1\} = \{01, 011\}$ and

$B.C = \{0,01\}.\{11\} = \{011, 0111\}$

So $(B.A) \cap \{B.C\} = \{01, 011\} \cap \{011, 0111\} = \{011\}$.

Hence $(B.A) \cap (B.C) = \{011\} \not\subseteq \emptyset = B.(A \cap C)$. So it is not always true that $(B.A) \cap (B.C) \subseteq B.(A \cap C)$. END