MAD 3512 - THEORY OF ALGORITHMS FLORIDA INT'L UNIV.

TEST #2 – Spring 2021 TIME: 75 min.

Answer all 6 questions. No calculators, notes, or on-line stuff are allowed. An unjustified answer will receive little or no credit. Draw a line to separate each of your 6 solutions to the 6 questions. (Double check the solutions that you send as a pdf file – to ensure it contains everything.)

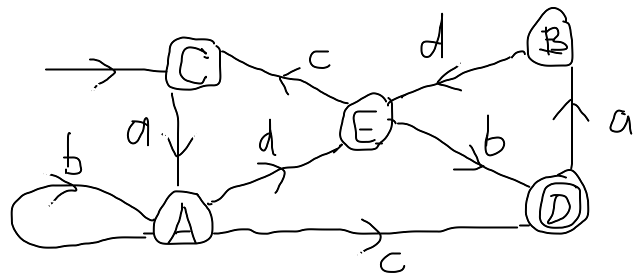
(16) 1. (a) Find an NFA, *M*, which is *equivalent* to the RLG *G* given below.

G: $\to D$, $D \to 01D$, $D \to 1C$, $C \to 10$, $C \to \lambda$, $C \to 1B$, $C \to A$, $B \to 1D$, $B \to \lambda$, $A \to 0C$, $A \to 11$.

(b) Find an RLG, *G*, which is equivalent to the NFA in Problem 2(a) below.

(16) 2. (a) Find a *regular expression* for the language accepted by the NFA *M* shown on the right.

(b) Define what is the *busy-beaver function*, $\beta(n)$.



(16) 3. (a) Define the *initial functions* and the operation called *primitive recursion*.

(b) Show that $F(x,y) = 4x+3y+2$ is a *primitive recursive function* by finding primitive recursive functions *g* and *h* such that $F = prec(g,h)$.

(16) 4.(a) What is the difference between a total function & a μ*-recursive* function on N.

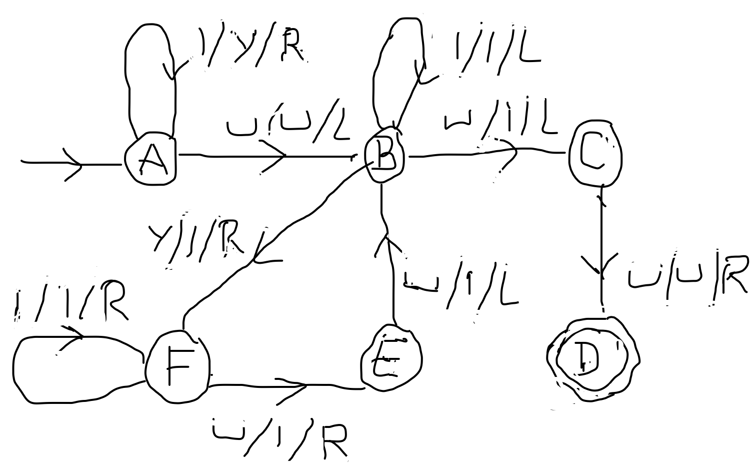
(b) Let $f(x)$= *Ceiling function* of $[(x^2+1)^{1/3}]$. Show that *f* is a μ*-recursive* function.

[*You may use the fact that PRED, MONUS, ADD, MULT & SIGN are primitive recursive if needed in #4, but you are* ***not*** *allowed to do so in Question #3.*]

(18) 5.(a) What is the difference between a *Turing-decidable language* and a *Turing semi-decidable language*.

(b) Show what happens at each step if (i) 1 and (ii) λ are the inputs for the TM, *M*, shown on the right.

(c) What is the *function computed* by *M* in monadic (base 1) notation?



(18) 6. Determine which of the following languages are regular and which are not.

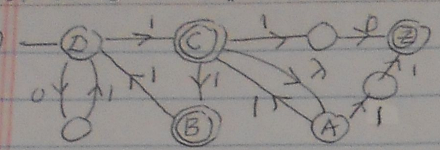
(a) $L_1 = \{a^k b^n : k \pmod 3 > (n^2 - 1) \pmod 3\}$ (b) $L_2 = \{b^k c^n : k > n^2 + 2\}$.

[*If you say that it is regular, you must find a regular expression for it; if you say it is non-regular, you must give a complete proof.*]

MAD 3512 – Theory of Algorithms Florida International Univ. (1)

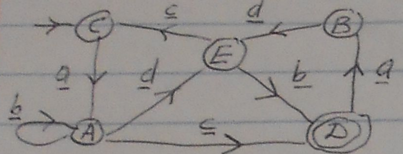
Solutions to Test #2 Spring 2021

#1(a)

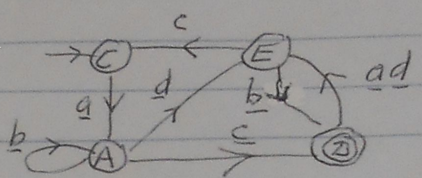


(b) G: $\to C$, $C \to aA$, $A \to bA$, $A \to dE$, $A \to cD$, $E \to cC$, $E \to bD$, $D \to \lambda$, $D \to bB$, $B \to dE$.

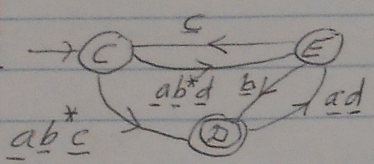
#2(a)

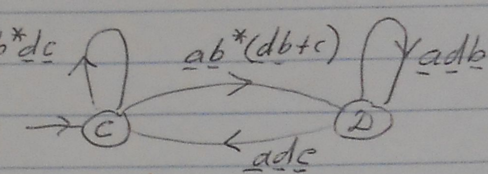


Elim. B



Elim. A





$L(M) = R_1^* R_2 (R_4 + R_3 R_1^* R_2)^* =$

$(ab^*dc)^* . ab^*(db+c) . [adb + adc . (ab^*dc) . ab^*(db+c)]^*$

2(b) $\beta(n)$ = max. no. of 1's a DTM in $\mathcal{H}_n$ can produce when started on the blank tape. $\mathcal{H}_n$ = set of all DTMs with n states + a halt state, & alphabet $\{1, \sqcup\}$ which halts when started on the blank tape.

#3(a) The initial functions are the constant 0, the zero function $Z(x) \equiv 0$, the successor function, $s(x) = x+1$, and the projective functions $I_{k,n}(\langle x_1, \dots, x_n \rangle) = x_k$ if $1 \le k \le n$ & λ if $k = 0$

Prim. recursion is the operation that takes two functions $g: \mathbb{N}^n \to \mathbb{N}$ & $h: \mathbb{N}^{n+2} \to \mathbb{N}$ & produces a function $f: \mathbb{N}^{n+1} \to \mathbb{N}$ by putting $f(\underline{x}, 0) = g(\underline{x})$ & $f(\underline{x}, s(y)) = h(\underline{x}, y, f(\underline{x}, y))$.

3(b) $f(x,y) = 4x + 3y + 2$. $f(x,0) = 4x + 2 \leftarrow g(x)$

$f(x, s(y)) = 4x + 3(y+1) + 2 = (4x+3y+2) + 3 = f(x,y) + 3 \leftarrow h(x, y, f(x,y))$

$\therefore h = s \circ s \circ s \circ I_{3,3}$ & $g(0) = 2$ & $g(s(y)) = 4(y+1) + 2 = g(y) + 4$

So $g = prec(s \circ s \circ 0,\ s \circ s \circ s \circ s \circ I_{2,2})$

$\therefore f = prec(g, h) = prec(prec(s \circ s \circ 0,\ s \circ s \circ s \circ s \circ I_{2,2}),\ s \circ s \circ s \circ I_{3,3})$

$g = prec(g_1, h_1)$ $g_1 = s \circ s \circ 0$, $h(\overset{I_{1,2}}{y}, \overset{I_{2,2}}{g(y)}) = g(y) + 4$ $h = s \circ s \circ s \circ s \circ I_{2,2}$

#4(a) A total function on is any function $g: \mathbb{N}^k \to \mathbb{N}$ which is defined for <u>all values</u> of $\underset{\sim}{x} \in \mathbb{N}^k$. A μ-recursive function is a <u>partial</u> function which can be obtained from the initial functions by a finite no. of cartesian products, prim. recursions compositions, and minimizations on total functions.

4(b) Let $g(x,y) = (x^2+1) \dot{-} y^3$. Then $f(x) = (\mu y)[g(x,y)=0]$

So $f = \mu[g, 0] = \mu[MONUS \circ (S \circ MULT \circ (I_{1,2} \wedge I_{1,2}) \wedge MULT(I_{2,2} \wedge MULT(I_{2,2} \wedge I_{2,2}))), 0]$

#5(a) A language L is Turing-decidable if we can find a TM M_1 such that M_1 <u>halts</u> in an accepting state on w, when $w \in L$ & M_1 <u>halts</u> in a non-accepting on w, when $w \notin L$.

L is Turing semi-decidable if we can find a TM M_2 such that M_2 halts in an accepting state on w, when $w \in L$ & M_2 fails to halt, <u>or</u> halts in a non-accepting state on w, if $w \notin L$.

5(b) $\vdash \langle A, \underline{1} \rangle \vdash \langle A, Y\underline{\sqcup} \rangle \vdash \langle B, \underline{Y}\sqcup \rangle \vdash \langle F\, 1, \underline{\sqcup} \rangle \vdash \langle E, 11\underline{\sqcup} \rangle \vdash \langle B, 1\underline{1}1 \rangle$

$\vdash \langle B, \underline{1}11 \rangle \vdash \langle B, \underline{\sqcup}111 \rangle \vdash \langle C, \underline{\sqcup}1111 \rangle \vdash \langle D, \underline{1}111 \rangle$ <u>halts</u>

(ii) $\vdash \langle A, \underline{\sqcup} \rangle \vdash \langle B, \underline{\sqcup}\sqcup \rangle \vdash \langle C, \underline{\sqcup}1\sqcup \rangle \vdash \langle D, \underline{1} \rangle$ <u>halts</u>

5(c) So $f(0)=1$, $f(1)=4$, [Check that $f(2)=7$.] $f(n)=3n+1$.

#6(a) If $n \equiv 0 \pmod 3$, then $k \pmod 3 > 0^2-1 \equiv 2 \pmod 3$, no value of k

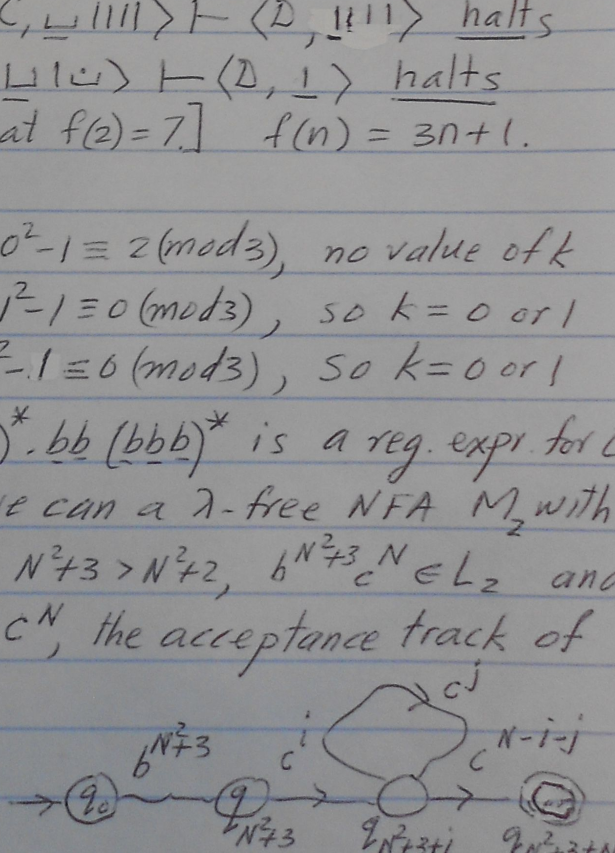
If $n \equiv 1 \pmod 3$, then $k \pmod 3 > 1^2-1 \equiv 0 \pmod 3$, so $k = 0$ or 1

If $n \equiv 2 \pmod 3$, then $k \pmod 3 > 2^2-1 \equiv 0 \pmod 3$, so $k = 0$ or 1

So $(\underline{\lambda}+\underline{a})(\underline{a}\,\underline{a}\,\underline{a})^* \underline{b}(\underline{b}\,\underline{b}\,\underline{b})^* + (\underline{\lambda}+\underline{a}).(\underline{a}\,\underline{a}\,\underline{a})^*.\underline{b}\,\underline{b}\,(\underline{b}\,\underline{b}\,\underline{b})^*$ is a reg. expr. for L_1

6(b). Suppose L_2 was reg. Then we can a λ-free NFA M_2 with N states such that $L(M_2) = L_2$. Since $N^2+3 > N^2+2$, $b^{N^2+3}c^N \in L_2$ and since it takes $N+1$ states to process c^N, the acceptance track of $b^{N^2+3}c^N$ must have loop as shown.

If we ride this loop twice, this shows that M_2 accepts $a^{N^2+3}c^{N+j}$.



But $N^2+3 \not> (N+j)^2+2 = N^2+2jN+j^2+2$. So this contradiction shows that L_2 is non-regular.