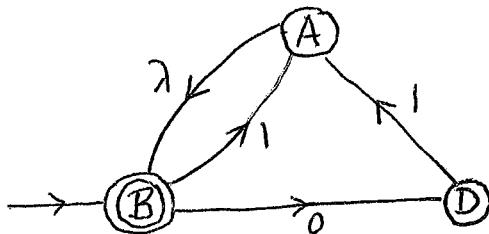


Answer all 6 questions. No calculators, notes, or on-line data are allowed. An unjustified answer will receive little or no credit. Begin each of the 6 questions on 6 separate pages.

- (15) 1.(a) Define what is the *reaching set* $R(\varphi)$ of a string φ in an NFA, M .
 (b) Let M be the NFA on the right. Find a DFA, M_C which *recognizes* $L(M)^C$.



- (15) 2. Find *regular expressions*, E_1 and E_2 , which describe the languages, L_1 & L_2 , below.
 (a) $L_1 = \{\varphi \in \{0,1\}^*: \varphi \text{ contains } \textbf{both } 110 \text{ and } 101 \text{ as substrings}\}$.
Indicate how 110111100 is described by E_1 by putting dots between characters.
 (b) $L_2 = \{\varphi \in \{a,b\}^*: \varphi \text{ contains } \textbf{at most one occurrence of the string } aa\}$.
Indicate how baabbab is described by E_2 by putting dots between the characters.

- (20) 3. (a) Define what it means for two states p & q to be *indistinguishable* in a DFA, M .
 (b) Check for *inaccessible* states, then *partition* the states of the DFA below into *blocks of indistinguishable states*, and find the equivalent *reduced machine*, M_R .

	A	→ B	C	D	E	F	G
0	B	G	D	B	B	A	F
1	F	B	F	C	G	C	E

- (15) 4. (a) Let $f(\varphi) = [3 + 2.n_a(\varphi) - n_b(\varphi)] \pmod{5}$. Find a DFA, M which accepts the language, $L_4 = \{\varphi \in \{a,b\}^*: f(\varphi) \text{ is } 0 \text{ or } 2 \pmod{5}\}$.
 (b) If $\varphi = babaa$ find $f(\varphi)$ & check that it agrees with your DFA with φ as input.

- (20) 5. (a) Find a *context-free grammar* G which generates the language
 $L_5 = \{a^n b^k : n \geq 3k+1 \text{ & } k \geq 0\} \cup \{d^n e^k : 0 \leq n \leq 2k+3 \text{ & } k \geq 0\}$.
 (b) Find *derivations* from your G for each of the strings: (i) $a^8 b^2$ and (ii) $d^5 e^2$.

- (15) 6. Let A, B, C , and D be languages that are based on the *alphabet* $\{0,1\}$.
 (a) Is it always true that $(A \cdot B) \cap (A \cdot C) \subseteq A \cdot (B \cap C)$?
 (b) Is it always true that $(B \cdot C) - (B \cdot D) \subseteq B \cdot (C - D)$? (*Justify your answers.*)

Solutions to Test #1

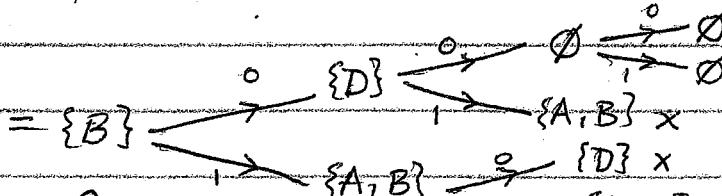
Spring 2022

1(a) The reaching-set $R(\varphi)$ is the subset of $Q(M)$ defined by

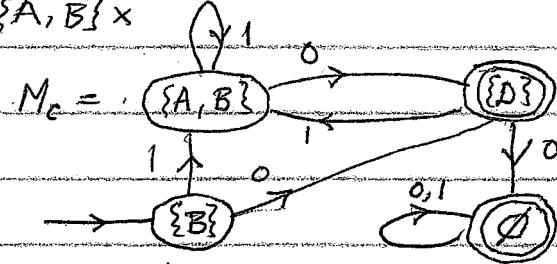
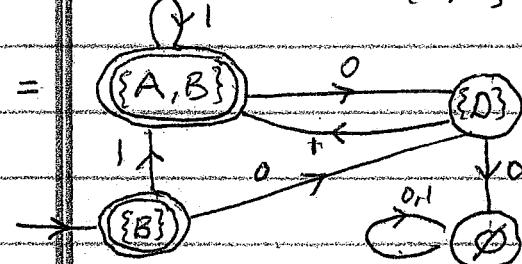
$$R(\varphi) = \{q \in Q(M) : \text{you can end-up at } q \text{ by starting at } q_0 \text{ & using } \varphi\}$$

(b)

$$R(\chi)$$



$$M_D =$$



$$2(a) E = (0+1)^*.101.(0+1)^*.100.(0+1)^* + (0+1)^*.110.(0+1)^*.101.(0+1)^*$$

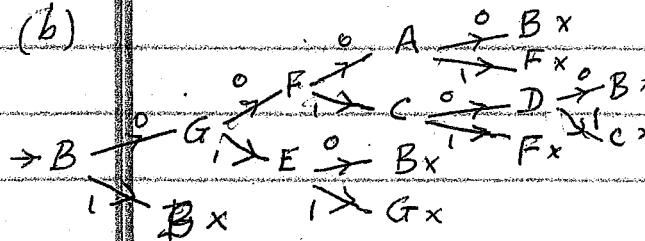
$$+ (0+1)^*.10110.(0+1)^* + (0+1)^*.1101.(0+1)^*$$

$$(b) (b+ab)^* + (b+ab).a + (b+ab).aa.(b+ba)^*$$

b, aa, b·ba·b

3(a) We say that p & q are indistinguishable in M whenever we have for each $\varphi \in T^*$, $\delta^*(p, \varphi) \in A(M) \Leftrightarrow \delta^*(q, \varphi) \in A(M)$.

(b)



$$\mathcal{P}_0: \{B, C, F\} \quad \{A, D, E, G\}$$

$$\mathcal{P}_1: \{B, C, F\} \quad \{A, D\} \quad \{E, G\}$$

$$\mathcal{P}_2: \{B\} \quad \{C, F\} \quad \{A, D\} \quad \{E, G\}$$

$$\mathcal{P}_3: \{B\} \quad \{C, F\} \quad \{A, D\} \quad \{E\} \quad \{G\}$$

$$\mathcal{P}_4: \{B\} \quad \{C, F\} \quad \{A, D\} \quad \{E\} \quad \{G\} = \mathcal{P}_3$$

(c)

$$M_R: \quad 1 \rightarrow \{B\} \quad \{C, F\} \quad \{A, D\} \quad \{E\} \quad \{G\}$$

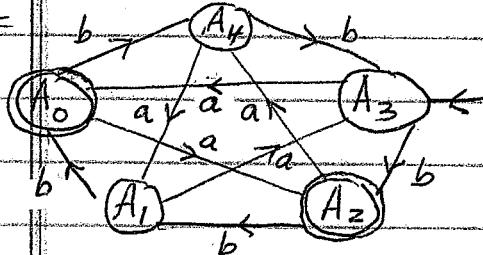
0	$\{G\}$	$\{A, D\}$	$\{B\}$	$\{B\}$	$\{C, F\}$
1	$\{B\}$	$\{C, F\}$	$\{C, F\}$	$\{G\}$	$\{E\}$

4(a) Let A_i ($i=0,1,2,3,4$) keep track of the fact that the part of the string processed so far is $i \pmod{5}$. Then A_0 & A_2 will be the accepting states and A_3 will be the initial state because $f(\lambda) = 3+0-0 \equiv 3 \pmod{5}$

$$\text{Also } f(\varphi a) = 3 + 2n_a(\varphi a) - n_b(\varphi a) = 3 + 2n_a(\varphi) - n_b(\varphi) + 2 = f(\varphi) + 2 \pmod{5}$$

$$f(\varphi b) = 3 + 2n_a(\varphi b) - n_b(\varphi b) = 3 + 2n_a(\varphi) - n_b(\varphi) - 1 = f(\varphi) + 4 \pmod{5}$$

So $M =$



$$(b) f(babaa) = 3 + 2(3) - 2$$

$$= 7 \equiv [2] \pmod{5}$$

$$\begin{array}{ccccccccc} b & a & b & a & a & a \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ A_3 & A_2 & A_4 & A_3 & A_0 & A_2 & \checkmark \end{array} \text{ Ans} = [2]$$

5(a) $S \rightarrow A/B$, $A \rightarrow aaaAb/C$, $C \rightarrow aC/a$, $B \rightarrow DDBe/DDD$,

(b)

$$(i) \rightarrow S \rightarrow A \rightarrow aaaAb \Rightarrow aaa \ a a a A b b \Rightarrow a^6 C b^2 \Rightarrow a^6 a C b^2 \Rightarrow a^6 a a b^2 = a^8 b^2$$

$$(ii) \rightarrow S \rightarrow B \Rightarrow DDBe \Rightarrow DDDDBee \Rightarrow DDDDDDDDee \Rightarrow D^6 dee$$

$$\Rightarrow D^5 ddee \Rightarrow D^4 dddee \Rightarrow D^3 ddddee \Rightarrow D^2 ddddee$$

$$\Rightarrow D \cdot \lambda \cdot d^5 e^2 \Rightarrow \lambda \cdot \lambda \cdot d^5 e^2 = d^5 e^2.$$

6(a) NO let $A = \{1, 10\}$, $B = \{1\}$, and $C = \{01\}$. Then

$$(A \cdot B) \cap (A \cdot C) = (\{1, 10\}, \{1\}) \cap (\{1, 10\}, \{01\}) = \{11, 101\} \cap \{101, 2001\} = \{101\},$$

$$A \cdot (B \cap C) = \{1, 10\} \cdot (\{1\} \cap \{01\}) = \{1, 10\} \cdot \emptyset = \emptyset. \text{ So in this}$$

case $(A \cdot B) \cap (A \cdot C) \neq A \cdot (B \cap C)$. Hence the result is not always true.

(b) YES. Let $\varphi \in (B \cdot C) - (B \cdot D)$. Then $\varphi \in (B \cdot C)$ and

$\varphi \notin (B \cdot D)$. So $\varphi = \beta \gamma$ with $\beta \in B$ and $\gamma \in C$,

and $\varphi = \beta \gamma \notin B \cdot D$. Now γ cannot be in D , otherwise $\beta \gamma$ would be in $B \cdot D$ which would contradict $\beta \gamma \notin B \cdot D$.

So $\gamma \in C$ and $\gamma \notin D$. Hence $\varphi = \beta \cdot \gamma \in B \cdot (C - D)$ because

$\beta \in B$ and $\gamma \in C - D$. So $\varphi \in (B \cdot C) - (B \cdot D) \Rightarrow \varphi \in B \cdot (C - D)$.

Hence we always have $(B \cdot C) - (B \cdot D) \subseteq B \cdot (C - D)$. END