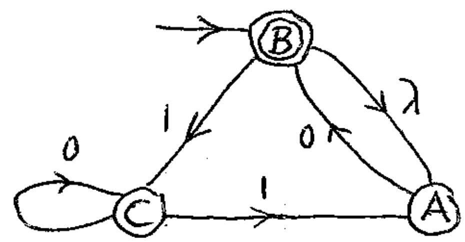
MAD 3512 - THEORY OF ALGORITHMS — FLORIDA INT'L UNIV.

TEST #1 - SPRING 2025 — TIME: 75 min.

Answer *all 6* questions. ***No calculators, notes, or on-line data are allowed.*** *An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON 6 SEPARATE PAGES.*

(15) 1.(a) Define what is a *regular expression over the alphabet* $V = \{a, b, d\}$.

(b) Let M be the *NFA* on the right. Find a *DFA*, M_C which *recognizes* $L(M)^C$.



(15) 2. Find *regular expressions*, E_1 and E_2, which describe the languages, L_1 and L_2, below.

(a) $L_1 = \{ \varphi \in \{0,1\}^* : \varphi$ contains both *001* and *010* as substrings $\}$.

Indicate how 101010010 is described by your E_1 by putting dots between characters.

(b) $L_2 = \{ \varphi \in \{b,c\}^* : \varphi$ contains *at most one* occurrence of the string *bb* $\}$.

Indicate how ccbcbbcbc is described by your E_2 by putting dots between characters.

(20) 3. (a) Define what it means for two states p & q to be ***indistinguishable*** in a *DFA*, *M*.

(b) Check for *inaccessible* states, then *partition* the states of the *DFA* below into *blocks of indistinguishable states,* and find the equivalent *reduced machine,* M_R.

	A	→B	C	D	E	F	G	H
0	F	H	D	F	E	G	C	F
1	G	C	B	B	H	B	A	C

(15) 4. (a) Let $f(\varphi) = [2.n_b(\varphi) - n_a(\varphi) - 1] \pmod 4$. Find a *DFA*, M_4, which accepts the language, $L_4 = \{ \varphi \in \{a,b\}^* : f(\varphi)$ is 1 or 2 $\pmod 4) \}$.

(b) If $\varphi =$ *babba* find $f(\varphi)$ & check that it agrees with your *DFA* with φ as input.

(20) 5. (a) Find a CFG (*context-free grammar*) G_5 which generates the language $L_5 = \{a^k b^n : n \geq 3k+2,\ k \geq 0\} \cup \{c^k d^n : 3 \leq n \leq 2k+4,\ k \geq 0\}$.

(b) Find *derivations* from your CFG for each of the strings: (i) a^1b^7 & (ii) c^2d^5.

(15) 6. Let A, B, C, and D be languages based on the *alphabet* $\{0,1\}$.

(a) Is it always true that $(B.A) - (C.A) \subseteq (B - C).A$?

(b) Is it always true that $(A.D) \cap (C.D) \subseteq (A \cap C).D$? (*Justify your answers.*)

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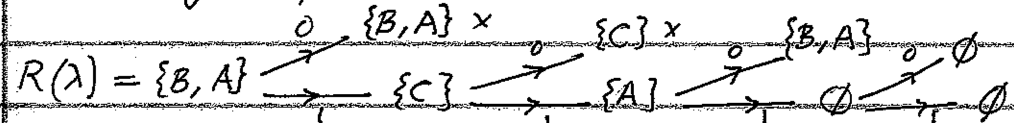
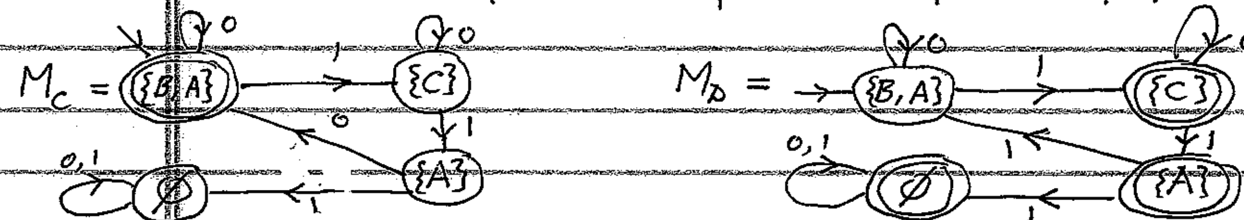
Solutions to Test #1 Spring 2025

#1 (a) A regular expression over $V=\{a,b,d\}$ is a finite string defined recursively as follows: (i) $\underline{a}, \underline{b}, \underline{d}, \underline{\lambda}$ and $\emptyset$ are regular expressions (ii) if E & F are reg. expressions, then so are $(E+F)$, $(E \cdot F)$, and (E^*).

(b)





#2 (a) $E_1 = (\underline{0}+\underline{1})^* . \underline{001} . (\underline{0}+\underline{1})^* . \underline{010} . (\underline{0}+\underline{1})^* + (\underline{0}+\underline{1})^* . \underline{010} . (\underline{0}+\underline{1})^* . \underline{001} . (\underline{0}+\underline{1})^*$
$+ (\underline{0}+\underline{1})^* . \underline{0010} . (\underline{0}+\underline{1})^* + (\underline{0}+\underline{1})^* . \underline{01001} . (\underline{0}+\underline{1})^*$

1 0 1 . 01001 . 0

(b) $E_2 = (\underline{c}+\underline{b}\underline{c})^* . (\underline{\lambda}+\underline{b}) + (\underline{c}+\underline{b}\underline{c})^* . \underline{b}\underline{b} . (\underline{c}+\underline{c}\underline{b})^*$

c . c . bc . bb . cb . c

#3. (a) Two states p & q are indistinguishable in a DFA M, if each $\varphi \in T(M)^*$, $\delta^*(p,\varphi) \in \mathcal{A}(M) \Leftrightarrow \delta^*(q,\varphi) \in \mathcal{A}(M)$.

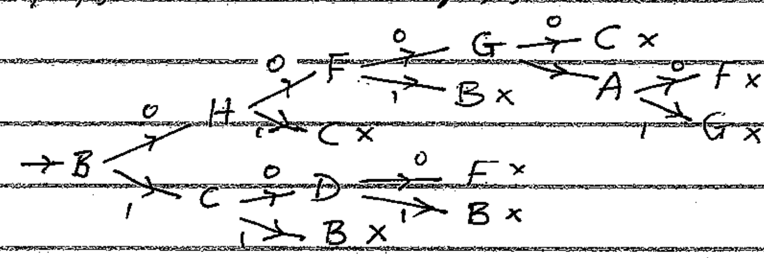
(b) $\mathcal{P}_0 = \{B,C,F\}, \{A,D,G,H\}$

$\mathcal{P}_1 = \{B,C,F\}, \{A,G\}, \{D,H\}$

$\mathcal{P}_2 = \{B,C\}\{F\}, \{A,G\}, \{D,H\}$

$\mathcal{P}_3 = \{B,C\}, \{F\}, \{A\}\{G\}, \{D,H\}$

$\mathcal{P}_4 = \{B,C\}, \{F\}, \{A\}, \{G\}, \{D,H\} = \mathcal{P}_3$



So E is inaccessible

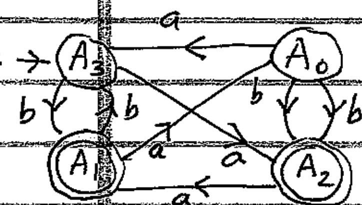
M_R:

	$\to\{B,C\}$	$\{F\}$	$\{A\}$	$\{G\}$	$\{D,H\}$
0	$\{D,H\}$	$\{G\}$	$\{F\}$	$\{B,C\}$	$\{F\}$
1	$\{B,C\}$	$\{B,C\}$	$\{G\}$	$\{A\}$	$\{B,C\}$

#4 (a) Let A_i $(i=0,1,2,3)$ keep track of the fact that the part of the string processed so far is $i \pmod 4$. Then A_1 & A_2 will be the accepting states. Since $f(\lambda) = 2n_b(\lambda) - n_a(\lambda) - 1 = 2(0) - 0 - 1 \equiv 3 \pmod 4$, A_3 will be the initial state.

#4(a) Also $f(\varphi a) = 2n_b(\varphi a) - n_a(\varphi a) - 1 = [2n_b(\varphi) - n_a(\varphi) - 1] - 1 \equiv f(\varphi) + 3 \pmod 4$
& $f(\varphi b) = 2n_b(\varphi b) - n_a(\varphi b) - 1 = [2n_b(\varphi) - n_a(\varphi) - 1] + 2 \equiv f(\varphi) + 2 \pmod 4$.

So

M = 

#4(b)

b	a	b	b	a	⊔
A_3	A_1	A_0	A_2	A_0	A_3 halt

check: $f(babba) = 2(3) - 2 - 1 \equiv 3 \pmod 4$

#5(a) $\to S$ (starting variable), $S \to A|C$ (this gives the union)
$A \to aAbbb|B$, $B \to Bb|bb$, (this gives $\{a^k b^k : n \geq 3k+2,\ k \geq 0\}$)
$C \to cCDD|dddD$, $D \to d|\lambda$ (this gives $\{c^k d^n : 3 \leq n \leq 2k+4,\ k \geq 0\}$.

(b)(i) $\to S \Rightarrow A \Rightarrow aAbbb \Rightarrow aBbbb \Rightarrow aBbb^3 \Rightarrow aBbbb^3 \Rightarrow abbb^5 = a^1 b^7$.

(ii) $\to S \Rightarrow C \Rightarrow cCDD \Rightarrow ccCDDD^2 \Rightarrow c^2 dddD\, DDD^2 \Rightarrow c^2 d^3 dD.D^3 \Rightarrow cd^3 dd\,DDD$
$\Rightarrow c^2 d^5 \lambda DD \Rightarrow c^2 d^5 \lambda\lambda D \Rightarrow c^2 d^5 \lambda\lambda\lambda = c^2 d^5$.

#6(a) YES. Let $\varphi \in (B.A) - (C.A)$. Then $\varphi \in B.A$ and $\varphi \notin C.A$
So $\varphi = \beta.\alpha$ for some $\beta \in B$ & $\alpha \in A$. Now β cannot be in C otherwise $\beta.\alpha$ would be in $C.A$ and this would contradict $\varphi = \beta.\alpha \notin C.A$. Hence $\beta \in B$ & $\beta \notin C$. So $\beta \in (B-C)$. Since $\alpha \in A$, it follows that $\varphi = \beta.\alpha \in (B-C).A$ So whenever $\varphi \in (B.A) - (C.A)$, $\varphi \in (B-C).A$. So it is always true that $(B.A) - (C.A) \subseteq (B-C).A$.

6(b) NO. Let $A = \{1\}$, $C = \{10\}$, and $D = \{1, 01\}$. Then
$A.D = \{1\}.\{1,01\} = \{11, 101\}$ & $C.D = \{10\}.\{1,01\} = \{101, 1001\}$.
So $(A.D) \cap (C.D) = \{11, 101\} \cap \{101, 1001\} = \{101\}$.
Also $(A \cap C).D = \emptyset.\{1,01\} = \emptyset$. Hence, in this case,
$\{A.D\} \cap \{C.D\} = \{101\} \not\subseteq \emptyset = (A \cap C).D$
So it is <u>not always true</u> that $(A.D) \cap (C.D) \subseteq (A \cap C).D$.

END