MAD 3512 - THEORY OF ALGORITHMS — FLORIDA INT'L UNIV.

SUPPLEMENTARY PROBLEMS

CHAPTER 1 - LANGUAGES & REG. EXPR.

E1. Let $A = \{a\}$ and $B = \{b\}$. Describe the following sets

(a) A^*

(b) $A^* . B^*$

(c) $(A.B)^*$

(d) $(A \cup B)^*$

(e) $(A \cap B)^*$

(f) $(A \cup B)^* . AB$

(g) $(A^* \cup AB)^* . A \quad \cup \ B^*$

(h) $(A^* \cup AB)^* - (BAB)^*$

E2 Find the simplest expressions for the following

(a) $\underline{\lambda} . \emptyset^*$

(b) $\underline{\lambda}^* . \emptyset^*$

(c) $A^* \cup \emptyset^*$

(d) $(\underline{\lambda} \cup A)^*$

(e) $(\emptyset \cup A)^*$

(f) $(\underline{\lambda}^* . \emptyset^*)^*$

(g) $\{\emptyset, \{\emptyset\}\} - \emptyset$

(h) $\{\emptyset, \{\emptyset\}\} - \{\emptyset\}$

(i) $\{\emptyset\} \cap \{\emptyset, \{\emptyset\}\}$

(j) $\{\emptyset\} \cap \emptyset$

(k) $\underline{\lambda} - \emptyset^*$

(l) $\underline{\lambda} - \{\emptyset^*\}$

(m) $\underline{\lambda}^* - \emptyset^*$

(n) $\underline{\lambda} . \emptyset$

E3 Find regular expressions for the following languages

(a) $\{\varphi \in \{0,1\}^* :$ 101 is a substring of $\varphi\}$

(b) $\{\varphi \in \{0,1\}^* :$ each 0 in φ is immediately followed by at least two 1's$\}$

(c) $\{\varphi \in \{0,1\}^* :$ each 1 in φ is immediately followed by the string 10$\}$

(d) $\{\varphi \in \{0,1\}^* :$ 101 is not a substring of $\varphi\}$

E4. Say which of the following equations are always true & provide proofs to justify your answers Also say which of the equations are not always true and provide counterexamples to justify your answers. A, B & C are arbitrary languages.

(a) $A(BA)^* = (AB)^*A$

(b) $(AB)^* = A^*B^*$

(c) $A.(B^* \cap C^*) = (A.B^*) \cap (A.C^*)$

(d) $A^*.B = B \cup A^*.AB$

(e) $(A \cap B)^* = A^* \cap B^*$

(f) $A^*.(B \cap C)^* = (AB \cup AC)^*$

(g) $(A^*B^*)^* = (A \cup B)^*$

(h) $A^*(B \cup C) = A^*.B \cup A^*.C$

E5. Let S be a set of strings of letters from the alphabet V. We say that S is commutative if for any α & β in S

$$\alpha . \beta = \beta . \alpha$$

(a) Prove that if $S \subseteq (\underset{\sim}{\omega})^*$, then S is commutative. Here ω = a fixed string and $(\underline{\omega})^* = \{\omega^n : n \geq 0\}$

(b) Prove that if S is commutative then we can find a string ω such that $S \subseteq (\underline{\omega})^*$

CHAPTER 4 - REGULAR LANGUAGES

E6. (a) Prove that the language $\{a^k b^{3k} : k \geq 1\}$ is non-regular

(b) Prove that the language $\{\varphi c \varphi : \varphi \in \{0,1\}^*\}$ is non-regular.

E7. A prime number is any positive integer $p > 1$ which has only 1 & p as its divisors. Let $L = \{a^p : p \text{ is a prime number}\}$. Prove that L is a non-regular language.

E8. Let X and Y be regular languages based on the alphabet V. Determine which of the following languages are always regular and which of them are not always regular.

(a) $\{w : w \in X \text{ and } w^R \in Y\}$

(b) $\{w : w \in X \text{ and } w^R \notin Y\}$

(c) $\{w : w \in X \text{ and } w^R = w\}$

(d) $\{\varphi : \varphi^R \in X \text{ or } \varphi \notin Y\}$

(e) $\{\varphi : \varphi^R \notin X \text{ and } \varphi \in X.Y\}$

E9. Determine which of the following languages are regular and which are non-regular

(a) $\{a^k c a^l c a^m : m = k+l\}$

(b) $\{a^k c a^l c a^m : m \equiv k+l \pmod 3\}$

(c) $\{a_1 \cdots a_{2n} \in \{0,1\}^* : a_1 \cdots a_n = a_{n+1} \cdots a_{2n}\}$

(d) $\{w \in \{0,1\}^* : N_0(w) - N_1(w)$ is an even positive integer$\}$

Here $N_0(w)$ = number of 0's in w.
and $N_1(w)$ = number of 1's in w.

E10. (a) Let $L = \{P 0^n Q 1^n R : n \geq 1\}$ where P, Q & R are non-empty languages based on $\{0,1\}$. Is there a choice of P, Q and R for which L is regular

(b) Find infinite sets $L_1 \subsetneq L_2 \subsetneq L_3$ such that L_1 & L_3 are non-regular and L_2 is regular.

(c) Find infinite sets $L_1 \subsetneq L_2 \subsetneq L_3$ such that L_1 & L_3 are regular and L_2 is non-reg.

E11. Suppose A & B are non-regular sets. Determine if it is possible for any of the following sets to be regular.

(a) $A - B$ (b) $A \cup B$ (c) $A . B$

CHAPTER 6 - RECURSIVE FUNCTIONS

E12. Define $\text{sign}(n) = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n>1 \end{cases}$ and

$$\overline{\text{sign}}(n) = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{if } n>1. \end{cases}$$

Show that sign & $\overline{\text{sign}}$ are primitive recursive

E13. Show that each of the following functions, that are defined below, are primitive recursive.

(i) $EXP(X,Y) = X^Y$

(ii) $ABS(X,Y) = |X-Y|$

(iii) $ZER(X) = \begin{cases} 1 & \text{if } X=0 \\ 0 & \text{if } X>0 \end{cases}$

(iv) $MIN(X,Y)$ = SMALLER OF X & Y

(v) $MAX(X,Y)$ = LARGER OF X & Y

(vi) $REM(X,Y)$ = Remainder after dividing Y by X

(vii) $QUO(X,Y)$ = quotient obtained by dividing Y by X

(viii) $EQU(X,Y) = \begin{cases} 1 & \text{if } X=Y \\ 0 & \text{if } X\neq Y. \end{cases}$

E14. Define $ls(X,Y) = \begin{cases} 1 & \text{if } X<Y \\ 0 & \text{if } X\geqslant Y \end{cases}$ and

$$gr(X,Y) = \begin{cases} 1 & \text{if } X>Y \\ 0 & \text{if } X\leq Y. \end{cases}$$

Show that ls and gr are primitive recursive functions.