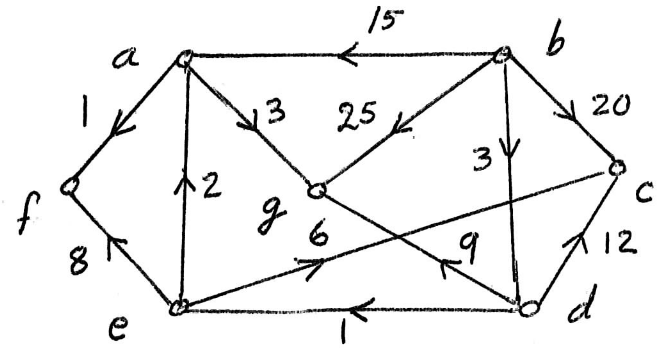
MAD 3305 - GRAPH THEORY FLORIDA INT'L UNIV.

TEST #1 - SPRING 2007 TIME: 75 min.

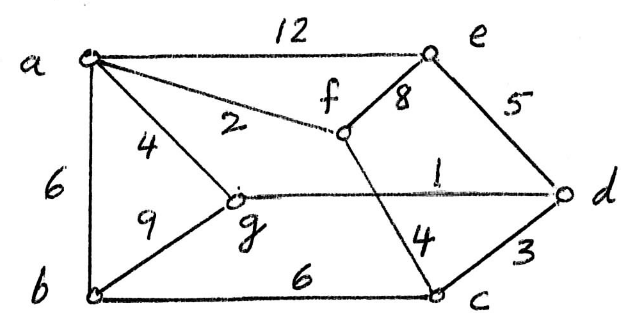
Answer *all* 6 questions. Provide all reasoning and show all working. *An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.*

(15) 1. Find the distances from *b* to each of the other vertices of the graph on the right by using *Dijkstra's Algorithm*.



(20) 2(a) Find a graph with degree sequence 5,3,3,3,3,3 by using the graphical sequence algorithm.

(b) Find a minimal spanning tree of the graph on the right by using *Prim's Algorithm* & starting at *b*.



(20) 3. (a) Find the tree corresponding to $\langle 2,1,2,6 \rangle$ via *Prufer's Tree Decoding Algorithm*.

(b) The six characters a,b,c,d,e,f occur with frequencies 25,4,6,15,40,10 respectively. Find an optimal binary coding for these six characters.

(15) 4. (a) Define what is a *connected component* of a disconnected graph G.

(b) Prove that if G is a disconnected graph, then G^c is forced to be connected.

(15) 5. (a) Define what is an *pendant vertex* of a graph G.

(b) Prove that in any non-trivial tree T the number of leaves is equal to

$$2 + \sum_{\deg(v)>2} \{\deg(v)-2\}$$

(15) 6. (a) Define what is a *legal flow* f in a network N and what is the *value* of the flow f.

(b) Prove that there are only two trees T such that T^c is also a tree.

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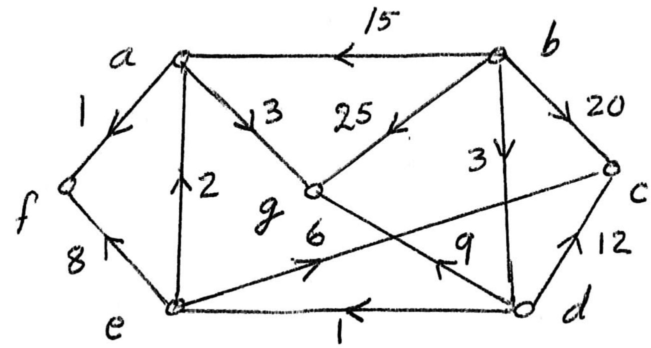

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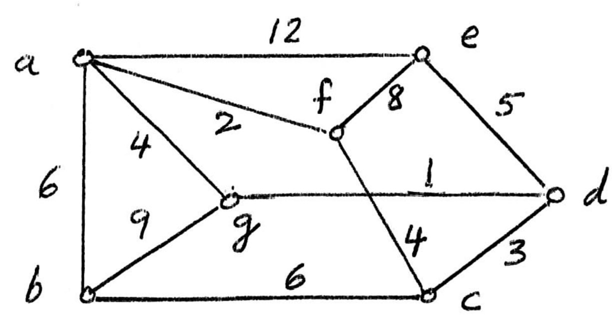

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MAD 3305 – Graph Theory
Solutions to Test #1

Florida Internat'l Univ.
Spring 2007.

1.

$L(a)$	$L(b)$	$L(c)$	$L(d)$	$L(e)$	$L(f)$	$L(g)$	T	v_0
∞	0	∞	∞	∞	∞	∞	$\{a,b,c,d,e,f,g\}$	b
15	.	20	3	∞	∞	25	$\{a,c,d,e,f,g\}$	d
15	.	15	.	4	∞	12	$\{a,c,e,f,g\}$	e
6	.	10	.	.	12	12	$\{a,c,f,g\}$	a
.	.	10	.	.	7	9	$\{c,f,g\}$	f
.	.	10	.	.	.	9	$\{c,g\}$	g
.	.	10	.	.	.	.	$\{c\}$	c
.	.	.	.	.	.	.	$\emptyset$	
6	0	10	3	4	7	9	$= d(b,\cdot)$	

2(a) 5, 3, 3, 3, 3, 3

2, 2, 2, 2, 2

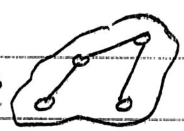
1, 1, 2, 2

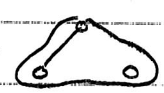
2, 2, 1, 1

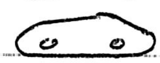
1, 0, 1

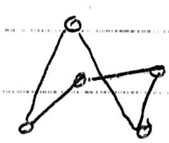
1, 1, 0

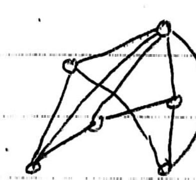
0 0

G_3 

G_2 

G_1 

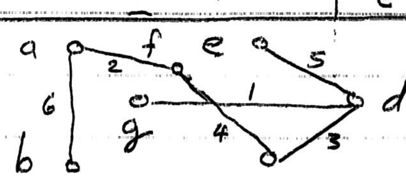
G_4 = 

G_5 = 

(b)

$E(T)$	U	a	b	c	d	e	f	g	x_0
		$d(U,\cdot)$							
$\emptyset$	$\{b\}$	6	.	6	∞	∞	∞	9	a
$\{ba\}$	$\{b,a\}$	.	.	6	∞	12	2	4	f
$\{ba, af\}$	$\{b,a,f\}$	.	.	4	∞	8	.	4	c
$\{ba, af, fc\}$	$\{b,a,f,c\}$	.	.	.	3	8	.	4	d
$\{ba, af, fc, cd\}$	$\{b,a,f,c,d\}$	.	.	.	.	5	.	1	g
$\{ba, af, fc, cd, dg\}$	$\{b,a,f,c,d,g\}$	.	.	.	.	5	.	.	e
$\{ba, af, fc, cd, dg, de\}$	$\{b,a,f,c,d,g,e\}$	STOP							

$\omega(T_{min}) = 21$.

T_{min} = 

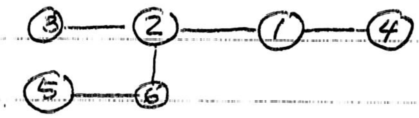
3(a) no. of edges x needs at stage i

X	i=1	i=2	i=3	i=4	
1	2	2 →	1 →	0	0
2	3 →	2	2 →	1 →	0
3	1 →	0	0	0	0
4	1	1 →	0	0	0
5	1	1	1	1	1
6	2	2	2	2 →	1

$|S| = 4 = p - 2 \Rightarrow p = 6$

i	j(i)	s(i)
1	3 —	2
2	4 —	1
3	1 —	2
4	2 —	6
plus	5 —	6

T =



(b) $\underbrace{4, 6}, 10, 15, 25, 40$

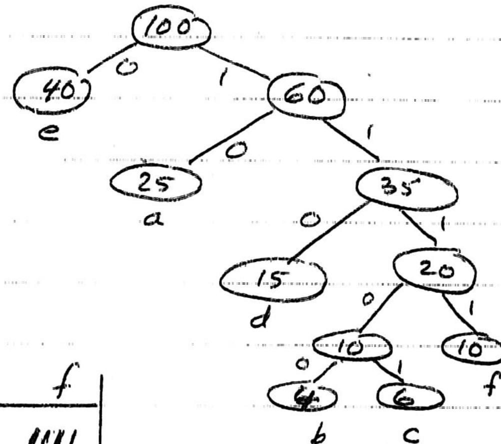
$\underbrace{10, 10}, 15, 25, 40$

$\underbrace{15, 20}, 25, 40$

$\underbrace{25, 35}, 40$

$\underbrace{40, 60}$

100



Char	a	b	c	d	e	f
Code	10	11100	11101	110	0	1111

4(a) A connected component of G is a maximal connected subgraph H of G. H is maximal in the sense that there is no connected subgraph H' which strictly contains H.

(b) Suppose G is disconnected. Let u & v be any two vertices in G^c. Now if $uv \notin G$, then $uv \in G^c$ and we instantly get a path $u - v$ from u to v in G^c. And if $uv \in G$, then there must exist a component H_2 of G that does not contain u & v. Choose a vertex w in H_2. Then uw & wv will both be edges in G^c. So $u - w - v$ will be a path from u to v in G^c. So in either case we get a path from u to v in G^c. Since u and v are arbitrary, it follows that G^c is a connected graph.

5(a) A pendant vertex of G is any vertex of degree 1.

(b) Let $p = |V(T)|$, $k =$ max. degree in T, and $n_i =$ number of vertices in T with degree i. Then $n_1 + n_2 + n_3 + \cdots + n_k = p$. Also sum of degrees in T $= 2(p-1)$. So : $1.n_1 + 2.n_2 + 3.n_3 + \cdots + k.n_k = 2p-2$

$\therefore \quad 1.n_1 + 2.n_2 + 3.n_3 + \cdots + k.n_k = (2n_1 + 2n_2 + 2n_3 + \cdots + 2n_k) - 2$

$\therefore \quad 2 + 0.n_2 + (3-2).n_3 + (4-2).n_4 + \cdots + (k-2).n_k = 2n_1 - n_1$

$\therefore \quad n_1 = 2 + (3-2).n_3 + (4-2).n_4 + \cdots + (k-2).n_k$

$$= 2 + \sum_{\deg(v) > 2} \{\deg(v) - 2\}$$

6(a) A legal flow in a network N is any function $f: E \to [0,\infty)$ such that (i) $f(e) \le c(e)$ for each $e \in E$, and (ii) $\sum_{e \in Out(v)} f(e) = \sum_{e \in In(v)} f(e)$ for each $v \in V - \{s,t\}$.
The value of the flow is defined by $F(f) = \sum_{e \in In(t)} f(e)$.

(b) Suppose T is a tree such that T^c is also a tree.
Let $p = |V(T)|$. Then $|E(T)| = |E(T^c)| = p-1$.
Also $|E(T)| + |E(T^c)| = |E(K_p)| = p(p-1)/2$.

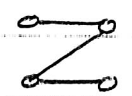
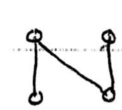
So $\quad (p-1) + (p-1) = p(p-1)/2$

$\therefore \quad 4(p-1) = p(p-1)$

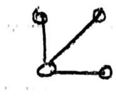
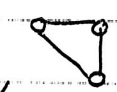
$\therefore \quad (4-p)(p-1) = 0 \quad \Rightarrow \quad p = 1$ or 4.

So there can only be two possible values of p.

If $p = 1$, then $T =$  and $T^c =$ 

If $p = 4$, then $T =$  and $T^c =$ 

So there are only two trees T such that T^c is also a tree.

Note : When $p = 4$, T cannot be  bec. $T^c =$  is not a tree.