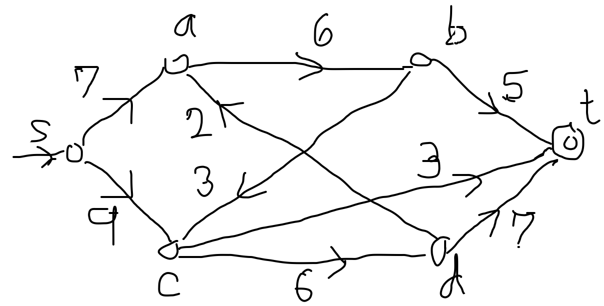
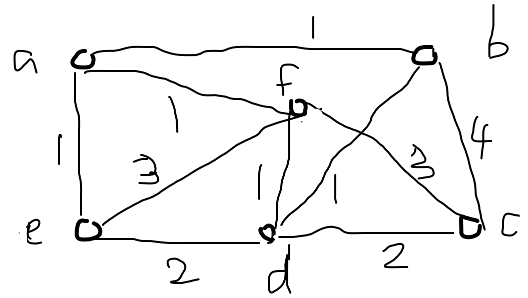
MAD 3305 - GRAPH THEORY FLORIDA INT'L UNIV.
TEST #2 - SPRING 2021 TIME: 75 min.

Answer all 6 questions. No calculators, notes, or on-line stuff are allowed. An unjustified answer will receive little or no credit. Draw a line to separate each of your 6 solutions to the 6 questions.
(Double check the solutions that you send as a pdf file – to ensure it contains everything.)

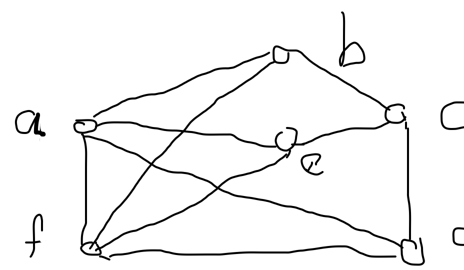
(15) 1. Find a maximal flow f^* in the network on the right by using the *Ford-Fulkerson Algorithm*. Also find the source-separating set of vertices S^* corresponding to f^*.



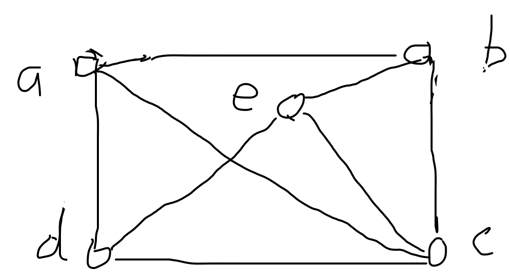
(15) 2. Find a *minimum postman walk* of the graph on the right by using the *Postman Algorithm*; and find *the total length* of your minimum postman walk?



(18) 3. Determine whether or not the graph on the right is planar by using the *DMP Planarity Algorithm*. [*Show the embeddings for each step of the algorithm*.]



(22) 4.(a) Find $P_G(\lambda)$ for the graph G on the right by using the *Chromatic Polynomial Algorithm.*
(b) Prove that $P_T(\lambda) = \lambda.(\lambda-1)^{n-1}$ for any tree T with n vertices.



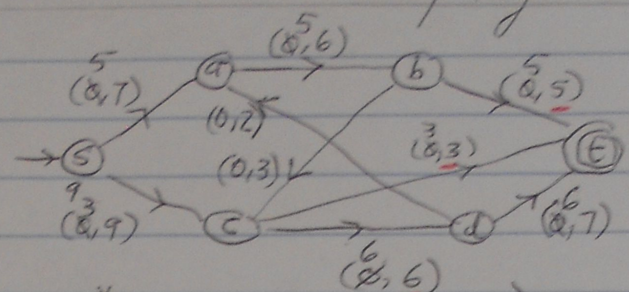
(15) 5.(a) Define what is a *Hamilton cycle* & what is an *Euler circuit* of a graph *G*.
(b) Write down what the *Euler's Circuit Theorem* says & use it to prove that any connected multi-graph with exactly *two odd vertices* has an *open Euler trail*.

(15) 6.(a) Define what is the *dual graph* G^* of a polyhedral graph *G* and define what is a *self-dual* polyhedral graph.
(b) Let G be a self-dual polyhedral graph with *p vertices and q edges*. Prove that $q = 2(p-1)$. Use this to find a self-dual polyhedral graph with 12 edges.
[*You may use any theorem that was proved in class for Qu. #6, if needed.*]

MAD 3305 - Graph Theory Florida Internat'l Univ.

Solutions to Test #2 Spring 2021

#1: 1st Aug. semi-path



$s \xrightarrow{0,9} c \xrightarrow{0,3} t$

Slacks 9 3 $\boxed{\mu_1 = 3}$

2nd Aug. semi-path

$s \xrightarrow{3,9} c \xrightarrow{0,6} d \xrightarrow{0,7} t$

Slacks 6 6 7 $\boxed{\mu_2 = 6}$

3rd Aug. semi-path

$s \xrightarrow{0,7} a \xrightarrow{0,6} b \xrightarrow{0,5} t$

Slacks 7 6 5 $\boxed{\mu_3 = 5}$

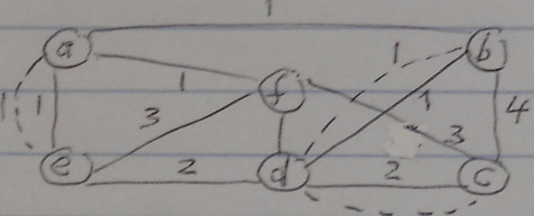
$S^* = \{s, a, b, c\}$ $c(S^*) = 5 + 3 + 6 = 14$

$Val(f^*)$ = net flow into t

$= 5 + 3 + 6 = 14 = c(S^*)$ ✓

#2

dist.	a	b	c	e
a	·	1		1
b		·	3	1
c			·	4
e				·



$\{a,b\}$ & $\{c,e\}$
$1 + 4 = 5$

$\{a,c\}$ & $\{b,e\}$
$4 + 1 = 5$

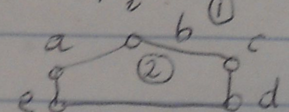
$\{a,e\}$ & $\{b,c\}$
$1 + 3 = 4$ ✓

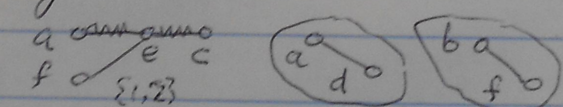
A Min. postman walk is of length $w(G) + 4 = 19 + 4 = 23$

$a \overset{1}{—} b \overset{4}{—} c \overset{2}{—} d \overset{2}{—} e \overset{3}{—} f \overset{3}{—} c \overset{2}{—} d \overset{1}{—} b \overset{1}{—} d \overset{1}{—} f \overset{1}{—} a \overset{1}{—} e \overset{1}{—} a.$

#3 i H_i Segments of G relative to H_i

1





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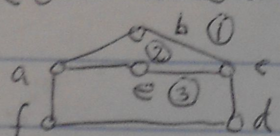

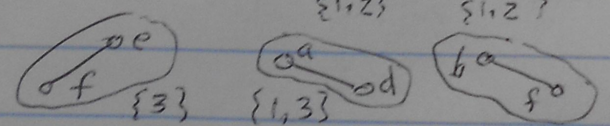

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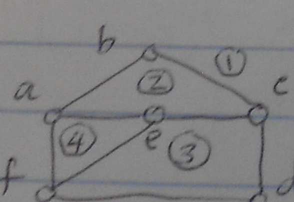

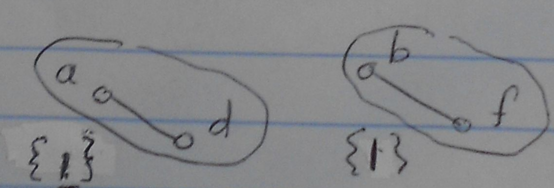

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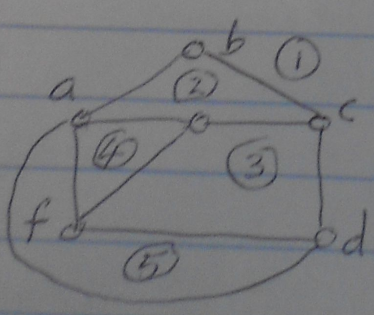
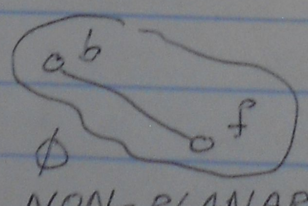
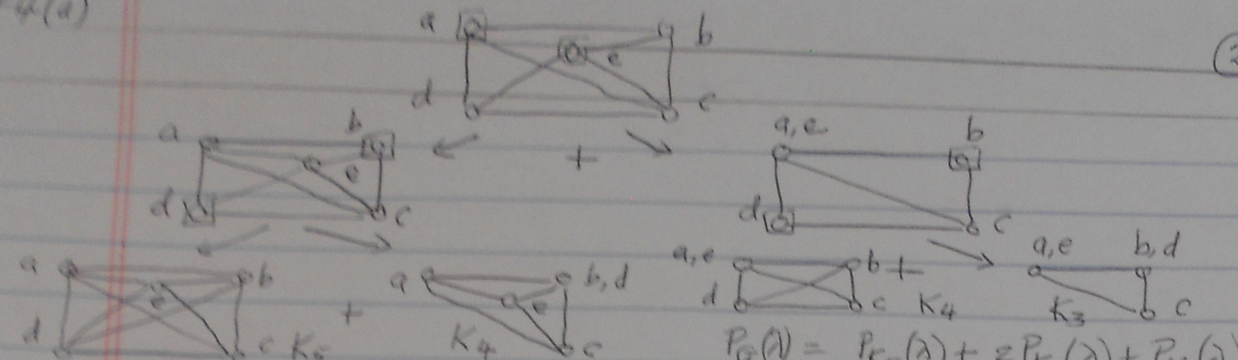




NON-PLANAR

#4(a)



$P_G(\lambda) = P_{K_5}(\lambda) + 2P_{K_4}(\lambda) + P_{K_3}(\lambda)$

$P_G(\lambda) = \lambda(\lambda-1)(\lambda-2)[(\lambda-3)(\lambda-4) + 2(\lambda-3)+1] = \lambda(\lambda-1)(\lambda-2)[\lambda^2-5\lambda+7]$.

#4(b) We will prove the result by induction on n. If $n=1$, then $T \cong K_1$, so $P_T(\lambda) = \lambda = \lambda(\lambda-1)^{1-1}$. So the result is true for $n=1$. Assume it is true for n. Let T be any tree with $n+1$ vertices. Since $n+1 \geq 2$, T is a non-trivial tree & so has a leaf v_0 (choose any maximal path in T, the endpoints will be leaves).
So $P_T(\lambda) = P_{T'}(\lambda).(\lambda-1) = \lambda(\lambda-1)^{n-1}.(\lambda-1) = \lambda(\lambda-1)^{n+1-1}$. $(T' = T-\{v_0\})$
So if the result is true for n, it will be true for $n+1$. Hence the result is true for all trees by Mathematical Induction.

#5(a) A Hamilton cycle of G is a <u>closed</u> walk ~~which~~ includes each vertex of G <u>exactly once</u>, except the first & last vertices are equal. An Euler circuit of G is a <u>closed</u> walk of G which includes each edge of G <u>exactly once</u>. <u>Euler's Circuit Theorem</u> says that a connected multi-graph G has an Euler-circuit $\Leftrightarrow$ each vertex of G is of even degree.

5(b) Let G be a connected multigraph with <u>exactly 2</u> vertices, v_0 and v_1 say, of odd degree. Put $G' = G \cup \{v_0v_1\}$. Then G' will have all vertices of even degree & so by Euler's Thm <u>G'</u> will have an Euler-circuit. If we remove the edge v_0v_1 from this trail, we will get an open Euler trail of G.

#6(a) The dual of G is defined by $\langle V(G^*), E(G^*)\rangle = G^*$, where $V(G^*)$ = set of faces of the polyhedron from which G came & each time two faces share an edge we get an edge in $E(G^*)$
G is self-dual if $G^* \cong G$.

6(b) $r(G) = q(G)+2-p(G)$ & $p(G^*) = r(G)$
So $p = q+2-p \Rightarrow q = 2(p-1)$.

$2(p-1)=12$
$\Rightarrow p=7$
$G \cong G^*$ $\therefore$ G is self-dual.

G =