MAD 3305 - GRAPH THEORY

SUPPLEMENTARY PROBLEMS

FLORIDA INT'L UNIV.

FALL 1992

S.1 (a) Let N_p be the graph with p vertices and no edges. Find $P(N_p, \lambda)$.

(b) Find $P(K_n - e, \lambda)$.

S.2 Find (a) $\chi(C_n)$, (b) $\chi(W_n)$

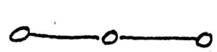
Here C_n is the cycle with n vertices and W_n is the wheel with n vertices.

S.3 (a) Suppose G is a bipartite graph with at least one edge. Prove that $\chi(G) = 2$

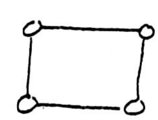
(b) If $\chi(G) = 2$, does it follow that G must be bipartite?

S.4 Find the chromatic number and chromatic polynomial of the following graphs

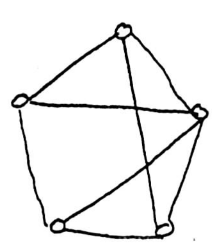
(a)



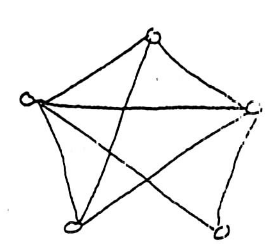
(b)



(c)



(d)



S.5 Let G be a connected graph with n vertices. Prove that $P(G, \lambda) \leqslant \lambda(\lambda - 1)^{n-1}$.

SUPPLEMENTARY HOMEWORK PROBLEMS

S.6 Find men-optimal and women optimal sets of stable marriages for the situation below.

MEN:	1st	2nd	3rd	4th
A	b	c	a	d
B	b	d	c	a
C	c	d	a	b
D	a	c	d	b

WOMEN:	1st	2nd	3rd	4th
a	A	D	C	B
b	C	D	B	A
c	D	B	A	C
d	B	A	D	C

S.7 Find a women-optimal set of stable marriages for the situation below.

MEN	1st	2nd	3rd	4th	5th
A	b	c	a	e	d
B	e	a	d	b	c
C	d	c	a	e	b
D	d	b	e	a	c
E	a	c	e	b	d

WOMEN	1st	2nd	3rd	4th	5th
a	D	B	A	C	E
b	B	C	A	E	D
c	C	D	B	A	E
d	D	B	A	E	C
e	B	D	A	E	C

S.8 Find women-optimal and men-optimal sets of stable marriages for the situation below.

MEN	1st	2nd	3rd	4th
W	A	C	D	B
X	D	A	B	C
Y	D	B	C	A
Z	B	D	C	A

WOMEN	1st	2nd	3rd	4th
A	Z	W	Y	X
B	W	Y	Z	X
C	Y	Z	X	W
D	Z	W	Y	X

ANSWERS TO SUPPLEMENTARY PROBLEMS

S.1. (a) $P(N_p, \lambda) = \lambda^p$

(b) $P(K_n - e, \lambda) = \lambda(\lambda-1)(\lambda-2)\cdots(\lambda-(n-3))\cdot(\lambda-(n-2))^2$

S.2 (a) $\chi(C_n) = \begin{cases} 2 & \text{if } n \text{ is even} \\ 3 & \text{if } n \text{ is odd} \end{cases}$

(b) $\chi(W_n) = \begin{cases} 3 & \text{if } n \text{ is odd} \\ 4 & \text{if } n \text{ is even} \end{cases}$

S.3 (a) Hint: Just color each partite set of the bipartite graph with the same color to get a legal coloring.

(b) Yes. Hint: Let V_i = set of all vertices receiving color i. V_1 and V_2 will form the partite sets of the bipartite graph.

S.4 (a) $\chi(G) = 2$, $P(G,\lambda) = \lambda(\lambda-1)^2$

(b) $\chi(G) = 2$, $P(G,\lambda) = \lambda(\lambda-1)(\lambda^2 - 3\lambda + 3)$

(c) $\chi(G) = 3$, $P(G,\lambda) = \lambda(\lambda-1)(\lambda-2)(\lambda^2 - 5\lambda + 7)$

(d) $\chi(G) = 4$, $P(G,\lambda) = \lambda(\lambda-1)(\lambda-2)^2(\lambda-3)$

S.5 HINT: Let T be a spanning tree of G. Then $P(T,\lambda) = \lambda(\lambda-1)^{n-1}$. Now think about what $P(G,\lambda)$ means and compare it with $P(T,\lambda)$.

ANSWERS TO SUPPLEMENTARY PROBLEMS

S.6 (a) Men-optimal set

A — c
B — b
C — d
D — a

(b) Women optimal set

a — A
b — C
c — D
d — B

Notice how much better the women did when they proposed!

S.7 Women optimal set:

a — E
b — A
c — C
d — D
e — B

Isn't it a shame that a got stuck with E!
On the plus side, however, E really likes a.

S.8 (a) Women-optimal set

A — W
B — Y
C — X
D — Z

(b) Men-optimal set

W — A
X — C
Y — D
Z — B

Notice again how the women did better when they proposed. Not all of them did better – some got the same choice – but some did do better!