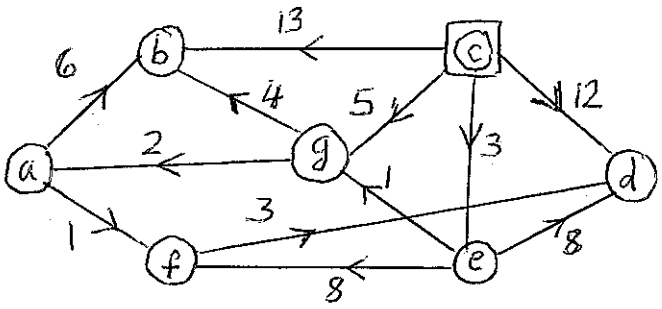
MAD 3301 - GRAPH THEORY NAME: _____.

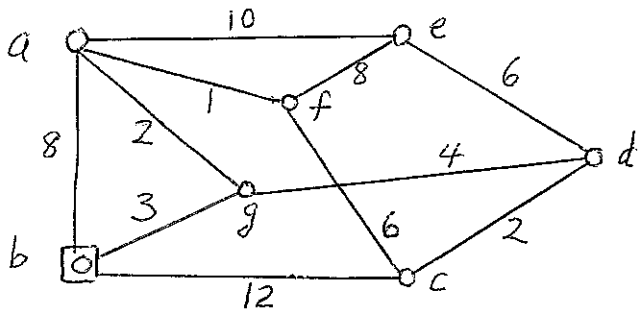
TEST #1 - FALL 2026 TIME: 75 min.

Answer *all 6* questions. ***No calculators, cell-phones, or notes are allowed.*** *An unjustified answer will receive little or no credit.* BEGIN EACH OF THE 6 QUESTION ON 6 SEPARATE PAGES.

(15) 1. Find the *distances* from *c* to each of the other vertices of the graph on the right by using *Dijkstra's Algorithm.*



(20) 2 (a) Find a *graph* with degree sequence ⟨5, 4, 3, 3, 2, 1⟩ by using *Graphical Sequence Algorithm.*
(b) For the graph on the right, find a *minimal spanning tree* by using *Prim's Algorithm* and starting at *b.*



(20) 3 (a) Find the *tree* that corresponds to the sequence ⟨2, 3, 4, 2⟩ via the *Prufer's Tree Decoding Algorithm.*

(b) The five characters *a, b, c, d, e* occur with frequencies *5, 12, 7, 20, 8;* respectively. Find an *optimal binary coding* for these five characters and the *weighted-path length* of your coding by using *Huffman's algorithm.*

(15) 4(a) Define (i) what is a ***tree,*** & (ii) what is a ***spanning-tree*** of a connected graph G.
(b) Prove any **tree T** with **p vertices** has p-1 edges. (c) Can a tree have ***one leaf***?

(15) 5 (a) Define what is the ***distance*** *d(u,v),* from *u* to *v* in a *weighted digraph* ⟨*G,w*⟩.
(b) If G is a *disconnected graph* with p vertices, prove that $|E(G)| \leq (p-1).(p-2)/2$.
(c) If $|E(G)| < (p-1).(p-2)/2$ & $|V(G)| = p$, is it still *possible* for G to be *connected*?

(15) 6 (a) Define what's a ***legal flow*** & the ***value*** *of a legal flow* in a network $N = \langle G,s,t,c \rangle$.
(b) A *tree T* has 6 vertices of deg. 5, 5 of deg. 4, 4 of deg. 3, & the rest of degree 1 or 2. How many *leaves* does T have? (c) What is the *smallest possible* value of $|E(T)|$?
[*You may use any theorems* ***proved in class*** *to answer question #6.*] END

MAD 3301 – Graph Theory — Florida Int'l Univ.

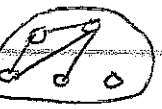
Solutions to Test #1 — Fall 2026

#1

	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	T(i)	i	v(i)
	∞	∞	0	∞	∞	∞	∞	{a,b,c,d,e,f,g}	0	c → b,d,e,g
	∞	13	.	12	3	∞	5	{a,b,d,e,f,g}	1	e → d,g,f
	∞	13	.	11	.	11	4	{a,b,d,f,g}	2	g → a,b
	6	8	.	11	.	11	.	{a,b,d,f}	3	a → b,f
	.	8	.	11	.	7	.	{b,d,f}	4	f → d
	.	8	.	10	.	.	.	{b,d}	5	b → ∅
	.	.	.	10	.	.	.	{d}	6	d → ∅
d(c,·)	6	8	0	10	3	7	4	∅ STOP		

#2. (a) ⟨5,4,3,3,2,1⟩

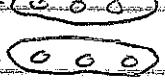
⟨3,2,2,1,0⟩ → G_3 =



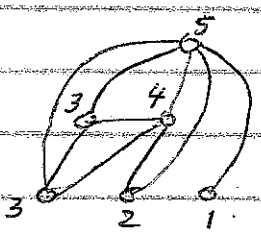
⟨1,1,0,0⟩ → G_2 =



⟨0,0,0⟩ → G_1 =

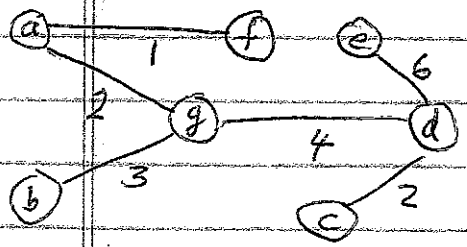


G_4 =



(b)

E(T)	V(T)	a	b	c	d	e	f	g	i	$X_0(i)$
∅	{b}	∞	0	∞	∞	∞	∞	∞	0	b → a,c,g
{bg}	{b,g}	8	.	12	∞	∞	∞	3	1	g → a,d
{bg, ga}	{b,g,a}	2	.	12	4	∞	∞	.	2	a → e,f
{bg, ga, af}	{b,g,a,f}	.	.	12	4	10	1	.	3	f → c,e
{bg, ga, af, gd}	{b,g,a,f,d}	.	.	6	4	8	.	.	4	d → c,e
{bg, ga, af, gd, dc}	{b,g,a,f,d,c}	.	.	2	.	6	.	.	5	c → ∅
{bg, ga, af, gd, dc, de}	{b,g,a,f,d,c,e}	.	.	.	.	6	.	.	6	e → ∅

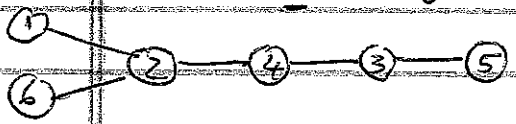


$w(T_{min}) = 3+2+4+1+2+6 = \boxed{18}$

These underlined numbers matches the weights of the minimal edges used.

3(a)

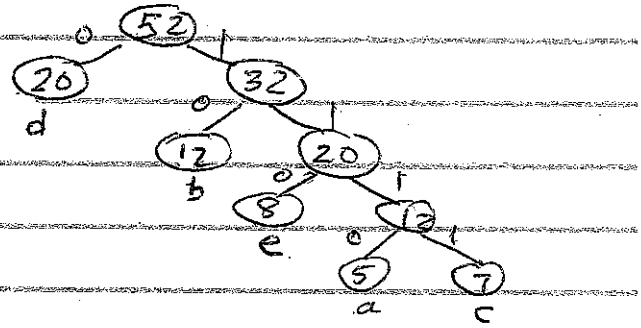
i	$d_i(1)$	$d_i(2)$	$d_i(3)$	$d_i(4)$	$d_i(5)$	$d_i(6)$	$\ell(i)$	$s(i)$
1	1	3	2	2	1	1	1 — 2	
2	0	2	2	2	1	1	5 — 3	
3	0	2	1	2	0	1	3 — 4	
4	0	2	0	1	0	1	4 — 2	
5	0	1	0	0	0	1	2 — 6	

$T =$ 

, $p = |5| + 2 = 6$

(b)

a	c	e	b	d
5	7	8	12	20
8	12	12	20	
12	20	20		
20	32			
52				



Char.	a	b	c	d	e
Freq.	5	12	7	20	8
Code	1110	10	1111	0	110

WPL(coding) $= 5(4) + 12(2) + 7(4) + 20(1) + 8(3)$
$= 20 + 24 + 28 + 20 + 24 = \boxed{116}$

4(a) A tree is any non-empty connected graph which contains no non-trivial cycles. A spanning tree of G is any subgraph H of G which is a tree and is such that $V(H) = V(G)$.

(b) We will prove that any tree with p vertices has $(p-1)$ edges by induction on p.. Basis: If $p=1$, then $|E(T)| = 0 = 1-1 = |V(T)|-1$. So the result is true for all trees with $p=1$.

Ind. step: Suppose the result is true for all trees with $\leq p$ vertices. Let T be any tree with $(p+1)$ vertices & e be any edge in T. Then $T-\{e\}$ will consist of two disjoint trees T_1 & T_2

So $|E(T)| = |E(T_1)| + |E(T_2)| + |\{e\}| = |V(T_1)|-1 + |V(T_2)|-1+1$

$= |V(T_1)| + |V(T_2)| - 1 = |V(T)| - 1$

. So if the result is true for all trees with $\leq p$ vertices, then it will be true for all trees with $p+1$ vertices. Hence the result follows by Induc.

5(a) $d(u,v) = \begin{cases} \text{length of the shortest directed path from } u \text{ to } v \text{ in } G, \\ +\infty, \text{ if there is no directed path from } u \text{ to } v \text{ in } G. \end{cases}$

(b) Suppose G is a disconnected graph with p vertices. Then we can split G into 2 disjoint parts G_1 & G_2 such that $G_1 \cup G_2 = G$ and $G_1 \cap G_2 = \emptyset$. Let $k = |V(G_1)|$. Then $|V(G_2)| = p-k$ & $|E(G)| \leq k(k-1)/2 + (p-k)(p-k-1)/2$. So $\{(p-1)(p-2)/2\} - |E(G)|$ $\geq [(p-1)(p-2) - k(k-1) - (p-k)(p-k-1)]/2 = [2pk - 2k^2 - 2p + 2]/2$ $= [pk - k^2 - p + 1] = (k-1)(p-k-1) \geq 0$ bec. $k \geq 0$ & $p-k-1 \geq 0$. Hence $[(p-1)(p-2)/2 - E(G)] \geq 0$ & so $|E(G)| \leq (p-1)(p-2)/2$.

(c) YES, just take $p=5$ & $G =$ . $|E(G)| = 4 \leq \frac{4(3)}{2} = 6$.

6(a) A legal flow in a network $N = \langle G, s, t, c \rangle$ is any function $f: E(G) \to [0, \infty)$ such that $f(e) \leq c(e)$ for $e \in E(G)$ & $\sum_{e \in In(v)} f(e) = \sum_{e \in Out(v)} f(e)$ for $v \in V(G) - \{s,t\}$. The value of f is defined by $Val(f) = \sum_{e \in In(t)} f(e) - \sum_{e \in Out(t)} f(e)$.

(b) Let $p = |V(T)|$, l = no. of leaves in T & k = no. of vertices of deg 2 in T. Then $p = 6+5+4+k+l = 15+k+l$ & $|E(T)| = p-1 = 14+k+l$. Now by a Theorem proved in class, sum of degrees in $T = 2|E(T)|$. So $6(5) + 5(4) + 4(3) + k(2) + l(1) = 2|E(T)| = 2.(14+k+l)$. $\therefore$ $30+20+12+2k+l = 28+2k+2l$. So $62 = 28+l$. $\therefore l = 34$. Hence T has 34 leaves.

(c) We know $|E(T)| = 14+k+l = 14+34+k = 48$. By merging-out all the vertices of degree 2 in T, we can get a tree T_{min} with only 48 vertices. So minimum value of $|E(T)|$ will be 48. END.

