MAD 3301 – GRAPH THEORY                      FLORIDA INT'L UNIV.
INFORMATION SHEET (Jan. 9th, 2024)                                        SPRING 2024

INSTRUCTOR: Prof. Ram, Office hours: 12:55pm-1:45pm and 3:30pm-4:45pm Tu &Th.

Prof Ram's homepage: <http://faculty.fiu.edu/~ramsamuj/> Prof Ram's e-mail: ramsamuj@fiu.edu

LAs Hours: Mr. Michael ILYIN (via Zoom) 10am-12pm Mon, 10am-11am Wed, & Fri. 5pm-8pm.

Mr. Michael ILYIN ZOOM’s Meeting ID:  **812 959 4200**   Code: **Mi2023** .

PREREQUISITE:  Discrete Math (MAD 2104) & COP 2210 (or its equiv.) – C grade or better.
A student needs a good working knowledge of  PROOFS  in order to succeed in this course.
OFFICIAL TEXTBOOK:    Graph Theory by Ronald Gould, Dover Publications, Inc. (2012).
This book is also available on line at:  <http://www.mathcs.emory.edu/~rg/>  & at my homepage.

SYLLABUS: Below are the relevant sections for about 96 % of the course.
             1. Fundamental Concepts:               Ch. 1 - Sec. 1, 2, 3, 4, 5, 6
             2. Connectedness & Distance:         Ch. 2 - Sec. 1, 2, 3, 4
             3. Trees and their uses:                    Ch. 3 - Sec. 1, 2, 3, 6
             4. Networks and Flows:                   Ch. 4 - Sec. 1, 2
             5. Edge and Vertex traversals:          Ch. 5 - Sec. 1, 2, 3, 6
             6.  Planar Graphs:                            Ch. 6 - Sec. 1, 2, 3
             7.  Graph Colorings:                        Ch. 8 - Sec. 1, 2, 5, 6
             8.  Matchings in Graphs:                  Ch. 7 - Sec. 1, 2

EXAM & ATTENDANCE  POLICIES:

 1. For each exam you are required to bring your Student ID and a blank blue exam booklet.

 (8"x11" available from FIU bookstore). No notes, calculators or cellphones are allowed.
 2. A **make-up test** for Exam #1 will be given - only if there is a verifiable case of illness or
      emergency**.**  If you miss Exam #2 for one of the same reasons that test will be discounted.

 (***see details below***)
 3. Any misconduct will be reported and dealt with according to the Code of Student Conduct.

 4. Sanctioned religious holidays can be accommodated with early notification (1st wk classes).

 5. Attendance & class participation (40 pts): 3 pts is deducted for each absence or missed exam.

 Attendance is mandatory. FIU will give you an automatic F for less than 60% attendance.

SCHEDULE OF EXAMS: No calculators, cell-phones, or notes are allowed in the exams.

Test #1 (**100** points): **THURSDAY, FEB. 22nd, 09:15am - 10:45am**

Test #2 **(100** points): **THURSDAY, APR. 11th, 09:15am - 10:55am**

Final Exam **(160** points): **TUESDAY, APR. 23rd, 09:30am - 12:00pm**\*

**(40** pts**)** Attendance & class participation: **3** pts is deducted for each absence or missed exam.

\* The final exam will be comprehensive - please note the time is earlier than the regular class.

GRADING SCHEME: The grades will be assigned as indicated below.

 F D D D C C C+ B- B B+ A- A

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 0% **48** 52 56 **60** 64 68 72 76 **80** 85 90 100%

 **0** **192pts** **240pts** 288pts **320pts** 360pts **400pts**

HOLIDAYS: Mon. Jan. 15th (M. L. King Day), Spring Break (Mon-Sat): Feb. 26th - Mar. 2nd.

DEADLINES: For add/drop (no liability) - Jan 16th. For DR/WI grade (no refund) - Mar. 18th.

**DETAILS FOR GETTING EXEMPT FROM AN EXAM & FOR GETTING AN “IN” GRADE.**

1. The first (and most important) rule at FIU is that each professor should treat every student in the same uniform way - unless we get documentation from an independent authority (such as the FIU Health Clinic, the FIU DRC, or an MD (medical doctor), etc.) or from a supervisor of the professor (such as the Division Chair-person, the Department Chair-person, or the Dean of the relevant College, etc.) to do otherwise. If the professor does not abide by this first rule, they can be fired.

2. So if a student is not feeling physically or mentally well before an exam, they should contact the FIU Health Clinic, or the FIU Counselling & Psychological Services (see below for the links), or your doctor – and get a signed letter that specifically says: ”Ms. X (or Mr. Y) is medically unable to take any exam on such and such a day. Please excuse Ms. X (or Mr. Y) from taking this exam on this specific day.” We are not authorized to know of the specific medical condition you may have. *You will lose 3 pts for being absent - because you did not come or participate in class*.

3. If this is the first exam in the course, the professor will attempt to give the student a make-up exam (if the student is able to take such an exam before the DR deadline) - so that the student can be have some feed-back about their performance before the deadline for getting a DR grade). Because of time constraints it is not possible to give a make–up exam for the second in-semester test before the final exam.

4. In order to get an “IN” grade (in which just the final exam needs to be completed during the next semester), a student must complete more than 50% of the graded material (i.e., the first two exams) and earn a passing grade so far. So if a student was exempted (for any of the reasons above) from the second exam, then that student does not qualify for an “IN” grade. Also if the student does not have a passing a grade when the first two exams are averaged, then that student does not qualify for an “IN” grade. This is an FIU rule – and all professors have to abide by it. We have to fill out a form and provide the documentation before any ”IN” grade can be entered.

5. If you missed the first test in the semester for an allowable, legitimate, verifiable reason you may be able to take a make-up exam as long as you provide adequate documentation and notify the professor at within two hours of the scheduled exam time. You must take the make-up test on the first opportunity you get to return to school. The allowable reasons are restricted to medical emergencies, traffic accidents in which your car becomes non-functional on your way to the exam, and deaths in the immediate family (parents, siblings, or children). We are not allowed to give make-up tests for work related issues. As always you must provide documentation. If you’re exempted from the 2nd exam, you will not get a make-up; your other scores will instead be used (with their corresponding weights) to produce your final grade – but in this case, please remember that you won’t qualify for an IN grade.

6. So please go & seek whatever medical treatment you may need- and get the necessary documentation - instead of telling the professor just a few minutes before the exam that you are unable to take it. Our hands are tied – we have to get documentation before we can give any make-up exam or exempt you from an exam. And if we give make-up exams or exemptions without documentation we can be fired and even be sued. If you have any questions about this matter, please contact the Math Division Director or the Math Undergraduate Program Director. Below are two useful links if you are not feeling well.

A. <https://studentaffairs.fiu.edu/health-and-fitness/student-health/>

B. <https://studentaffairs.fiu.edu/health-and-fitness/counseling-and-psychological-services/>

MAD 3305 - GRAPH THEORY                                            FLORIDA INT'L UNIV.
HOMEWORK SHEET

PRE-REQUISITE: Discrete Math (MAD 2104 or MAA 3200) & COP 2210 – C grades or better.
A student needs a good working knowledge of proofs  to succeed in this course.

OFFICIAL TEXTBOOK:    Graph Theory by Ronald Gould (Dover 2012 edition)
The textbook is also available online at  <http://www.mathcs.emory.edu/~rg/>  & at my homepage.

This is a list of the problems for the course.  As the semester proceeds we may need to add a few

more problems or to delete a few problems.  During classes a few more problem that are closely

related to the material discussed will also be assigned.   It is important that you do all the

assigned problems if you want to successfully complete the course.

Ch.1   Nos     1, 5, 6, 7 (except the part about G1[G2]), 9, 10, 12,

13, 14, 16, 17, 18, 19, 20, 23, 24, 25, 27, 28, 29.

Ch.2    Nos      1, 8, 9, 11, 16, 20, 21, 22, 24, 25, 29, 30, 31, 32.

Ch.3    Nos      1, 2, 3, 5, 6, 8, 9, 15, 16, 24.

Ch.4    Nos      1, 5, 9.

Ch.5    Nos      1, 2, 4, 10, 11, 15, 16, 17, 18, 19, 41.

Ch.6    Nos      1, 2, 5, 7, 8, 9, 10, 11, 12, 14, 16.

Ch.8    Nos      4, 5, 10, 27 and

 Nos  1, 2, 3, 4, 5  from the supplementary problems.

Ch.7    Nos      6, 7, 8  from the supplementary problems.

 Prof. Ram's Homepage: <http://faculty.fiu.edu/~ramsamuj/>

Solutions to the textbook's HW Problems & the Suppl. HW problems (37 pages)

[Ch. 1 & 2 (p. 1-13)](http://www2.fiu.edu/~ramsamuj/graphtheory/Graph_Sol_Ch1-2.pdf)        [Ch. 3 & 4 (p. 14-21)](http://www2.fiu.edu/~ramsamuj/graphtheory/Graph_Sol_Ch3-4.pdf)      [Ch. 5, 6 & 8 (p. 22-37)](http://www2.fiu.edu/~ramsamuj/graphtheory/Graph_Sol_Ch5-6%268.pdf)

8 Supplementary Problems & their Solutions (4 pages)

[Actual Homework Questions from the text (14 pages)](http://faculty.fiu.edu/~ramsamuj/graphtheory/Graph_HW_Quest.pdf)   (End of HW problems)

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MAD 3301 - GRAPH THEORY FLORIDA INT'L UNIV

REVISION FOR TEST #1 REMEMBER TO BRING AN 8x11 BLUE EXAM BOOKLET

KEY DEFINITIONS AND MAIN CONCEPTS

 Digraph, multi-digraph, pseudo-digraph, multi-pseudo-digraph (digraph-like object), in-degree & out-degree, Graph, multi-graph, pseudo-graph, multi-pseudo-graph (graph-like object), degree of a vertex, degree sequence, (G), Δ(G), sub-graphs, , regular graphs, adjacency matrix; geometric, set-theoretic, and matrix representation; walk, trail, circuit, cycle, path, distances in weighted graphs; inaccessible vertices, eccentricity, diameter, center, & radius of a graph; connected graphs, the connected components of a graph, weakly & strongly connected digraphs, bridge (cut-edge), edge connectivity, vertex connectivity, cut-vertex, pendant vertices, trees, forests, non-identical trees, leaves, minimum spanning trees, rooted trees, levels, height of a tree, children, parent, ancestors, descendants, n-ary trees, binary trees, codes, coding, uniquely decipherable coding, optimal coding, weighted path-length of a coding, [~~pre-order & post-order traversals, prefix & postfix notations~~], Networks, source & sink, capacity of an edge, source & sink, capacity constraint, conservation of flow, legal flow, value of a flow, source separating set of vertices, capacity of a source separating set of vertices, *MAXFLOW (N), MINCUT(N)*.

MAIN ALGORITHMS & PROBLEM SOLVING TECHNIQUES

1. (a) Graphical sequence algorithm

 (b) Graph recovery algorithm,

2. (a) Dijkstra’s distance algorithm,

 (b) Dijkstra’s shortest-path algorithm [modification of 2(a)]

3. (a) Kruskal's minimum-weight spanning tree algorithm,

 (b) Prim's minimum-weight spanning tree algorithm,

4. (a) Prufer's tree-encoding algorithm,

 (b) Prufer's tree-decoding algorithm,

5. Huffman's optimal-coding algorithm.

6. Finding the number of leaves or the minimum number of vertices in certain trees.

MAIN THEOREMS

1. The decreasing sequence <a,d2,d3, ... ,dp > is graphical if and only if

 <d2-1,d3-1,...,da+1-1,da+2,...,dp> is graphical . (*Graphical Sequence Theorem*)

2. The number of walks of length n from vi to vj = (An) [i, j].

3. (a) A connected graph with p vertices has at least p-1 edges.

 (b) A graph with p vertices and more than (p-1)(p-2)/2 edges is always connected.

4. (a) If G is a disconnected graph, then Gc must be connected.

 (b) If G has p vertices and (G) > (p-1)/2, then G is connected.

5. (a) Any tree with p vertices has exactly p-1 edges.

 (b) G is a tree if and only if there is exactly one path between any two vertices.

6. In any n-ary tree T with p vertices we have logn [{p.(n-1) +1}/n] < h(T) < p-1.

7. There are pp-2 different (non-identical) trees on p distinct vertices. (*Cayley's theorem*)

MAD 3301 - GRAPH THEORY FLORIDA INT'L UNIV

REVISION FOR TEST #2 REMEMBER TO BRING AN 8x11 BLUE EXAM BOOKLET

KEY CONCEPTS AND MAIN DEFINITIONS:

Networks, source & sink, capacity of an edge, legal flow, value of a flow, source-separating set of vertices, cut associated with a source-separating set, capacity of a cut, augmenting semi-paths, Euler circuits, Open Euler trails, Chinese postman problem, minimum postman walk, Hamilton cycles, Hamilton paths, Ore-type graphs, traveling salesman problem, minimum salesman walk, Hamilton-connected graphs, planar graphs, planar embeddings, maximal planar graphs, K5, K3,3, pieces of the first and second kinds, segments, embeddability of a segment in a region, non-separable blocks, polyhedral graphs, the five regular polyhedra, creating & merging out vertices of degree 2, shrinking edges, homeomorphisms, geometric dual, self-dual graphs, dual polyhedra, legal colorings, chromatic number, chromatic polynomial, modified chromatic algorithm, planar maps, the five color problem, the four-color problem, maps on other surfaces, the torus, the pretzel, [matchings in graphs & bipartite graphs, Stable marriages, Room-mate problem] .

MAIN ALGORITHMS:

 1. Ford & Fulkerson’s maximal-flow algorithm.

 2. (a) Fleury's Euler-circuit (& open Euler-trail) algorithm.

 (b) Minimal postman-walk algorithm (Chinese postman algorithm).

 3. Pre-processing graphs for planarity & the DMP planarity algorithm.

 4. Chromatic polynomial algorithm & the Modified chromatic polynomial algorithm.

 [5. Gale & Shapley’s stable marriage algorithm.]

MAIN THEOREMS:

 1. The maximum possible value of a flow in a network is equal to the minimum capacity of the cuts which separate the source and sinks. (*MaxFlow-MinCut Theorem*)

 2. (a) The connected graph G has an Euler circuit iff each vertex in G is of even degree.

 (b) It has an open Euler trail if and only if G has exactly two vertices of odd degree.

 (*Euler's Theorem*)

 3. (a) If deg(x) + deg(y) > p for all pairs of non-adjacent vertices x & y in G and p > 3, then G has a Hamilton cycle. (b) If deg(x) + deg(y) > p-1 for all pairs of non-adjacent vertices x & y in G, then G has a Hamilton path. (*Ore's Theorem*)

 4. (a) If G is a connected planar graph, then r = q+2-p. (*Euler's formula*)

 (b) If G is a planar graph with k components, then r = q+k+1-p. (*Gen. Euler's formula*)

 5. (a) Every region of a maximal planar graph with p > 3 is bounded by 3 edges.

 (b) In any maximal planar graph with p > 3 vertices, we have q = 3p-6.

 6. G is planar if an only if G has no subgraph which is homeomorphic to K5 or K3,3 . (*Kuratowski's Theorem*)

 7. (a) If a and b are non-adjacent vertices , then P(G, ) = P(G+ab, ) + P(Gab, ).

 (b) In any graph G, (G) < (G) + 1. (Here (G) = largest degree in G.)

[8. In a bipartite graph with partite sets X & Y, X can be matched with a subset of Y iff

 |N(S)| > |S| for all S X. (*P. Hall's Marriage Theorem*) ]