

Ch.3.1 #1(a) n is an integer larger than 1 ; n is not prime

(b) $2^{15}-1$ is not prime ; $2^{15}-1 = 7 \cdot 4681 = 31 \cdot 1057$

(c) Nothing, ($n=11$ is not prime, so the hypotheses are not satisfied)
but $2^{11}-1=23 \cdot 89$ tells us the converse of the Theorem is false.

#3(a) n is a natural number larger than 2 ; n is not prime

(b) Hint: Take $n=13 \cdot 2$. Then $2n+13=2 \cdot 13 \cdot 2+13=5 \cdot 13$

#4. Hint : $b^2-a^2=(b-a)(b+a)$

Now use the fact that $b-a > 0$ and $b+a > 0$.

#8. We want to show that $x \notin D \rightarrow x \in B$. It will suffice to show that $x \notin B \rightarrow x \in D$.

So suppose $x \notin B$. Then

$x \in A - B$ because $x \in A$ (given) & $x \notin B$

Since $A - B \subseteq C \cap D$, it follows that $x \in C \cap D$.

So $x \in C$ and $x \in D$. Hence $x \in D$.

#9. Hint : $\frac{a+b}{2} < b + b$ because $a < b$

$$\therefore \frac{a+b}{2} < \frac{b+b}{2}$$

#10. Hint: Use the contrapositive. Prove that if $x=8$, then $(\sqrt[3]{x}+5)/(x^2+6) \neq 1/x$.

#11 Hint: Suppose $ac \geq bd$. Then $ac/b \geq d$

$$c = \frac{b}{b} \cdot c > \frac{a}{b} c \quad \text{because } b > a$$

$$\geq d$$

$$\therefore c > d.$$

A(14)

Ch.3.1 #12 Suppose $x > 1$. Then

$$\begin{aligned} 2y &\leq 5 - 3x && \text{because } 3x + 2y \leq 5 \\ \therefore 2y &< 5 - 3x && \text{because } x > 1 \\ \text{So } 2y &< 2. \quad \text{Hence } y < 1. \end{aligned}$$

#14 Suppose $x > 3$ and $y < 2$. Then

$$\begin{aligned} x^2 - 2y &> 3^2 - 2y && \text{because } x > 3 \text{ so } x^2 > 3^2 \\ &> 3^2 - 2(2) && \text{because } y < 2 \\ &= 5 \end{aligned}$$

$\therefore x^2 - 2y > 5.$

Ch.3.2 #1. We are given that $P \rightarrow Q$ and $Q \rightarrow R$ are true.

We want to prove that $P \rightarrow R$ is true. So suppose P is true. Then Q will be true because $P \rightarrow Q$ is true. Now since Q is true and $Q \rightarrow R$ is true, it follows that R is true. Hence $P \rightarrow R$ is true.

#3. We are given that $A \subseteq C$ and $B \cap C = \emptyset$.

Suppose $x \in A$. Then $x \in C$ because $A \subseteq C$.

Now x cannot be in B , otherwise $x \in B$ and $x \in C$ so $B \cap C$ will not be empty. Hence $x \notin B$.

Thus $x \in A$ implies $x \notin B$.

#5. We are given that $A \cap C \subseteq B$ and $a \in C$. Suppose that $a \in A - B$. Then $a \in A$ and $a \notin B$. Now $a \in A$ and $a \in C$. So $a \in A \cap C$. Thus $a \in B$ because $A \cap C \subseteq B$. But this contradicts the fact that $a \notin B$. Hence $a \notin A - B$.

A(15)

Ch. 3.2 # 6 We are given that $A \subseteq B$, $a \in A$, and that a and b are not both elements of B . Suppose $b \in B$. Since $a \in A$ and $A \subseteq B$, it follows that $a \in B$. So we now have that both a and b are elements of B - a contradiction. Hence $b \notin B$.

#13 Modus tollens says: From $P \rightarrow Q$ and $\neg Q$, infer $\neg P$. Just check that $[(P \rightarrow Q) \wedge (\neg Q)] \rightarrow (\neg P)$ is a tautology as follows:

<u>P</u>	<u>Q</u>	<u>$[(P \rightarrow Q) \wedge (\neg Q)]$</u>	<u>\rightarrow</u>	<u>$(\neg P)$</u>
T	T	T	T	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

#17. No. Let's try. Suppose $y=4$. (We will try to get a contradiction and then conclude $y \neq 4$). Then from $x^2+y=13$, we get $x^2+4=13$. So $x^2=9$.

Now we would like to conclude that $x=3$ (to contradict the fact that we were given $x \neq 3$) but we can't do so. It is possible for x to be -3 .

We actually got a counter-example to the supposed theorem : If $x^2+y=13$ and $x \neq 3$, then $y \neq 4$.

Just take $x=-3$ and $y=4$. Then the hypotheses are satisfied but the conclusion does not hold.

$$\begin{aligned}
 \text{Ch. 3.3 #1 } (\exists x) [P(x) \rightarrow Q(x)] &\Leftrightarrow (\exists x) [\neg P(x) \vee Q(x)] & A(16) \\
 &\Leftrightarrow (\exists x) (\neg P(x)) \vee (\exists x) Q(x) \\
 &\Leftrightarrow \neg [(\forall x) (P(x))] \vee (\exists x) Q(x) \\
 &\Leftrightarrow [(\forall x) P(x)] \rightarrow (\exists x) Q(x)
 \end{aligned}$$

2 We want to show that $(\forall x) [x \in A \cap B \rightarrow x \in C]$.

Let x be an arbitrary element of $A \cap B$. Then $x \in A$ and $x \in B$. Now suppose $x \notin C$. Then $x \in A$, and $x \in B$ and $x \notin C$. So $x \in A$ and $x \in B - C$. So $x \in A \cap (B - C)$. So $A \cap (B - C) \neq \emptyset$, contradicting the fact that A and $B - C$ are disjoint. Hence $x \in C$. Thus if $x \in A \cap B$, then $x \in C$. Since x was arbitrary it follows that $A \cap B \subseteq C$.

4. We want to show that $(\forall B) [B \subseteq A \rightarrow B \subseteq P(A)]$
This means that for an arbitrary B , $B \subseteq A \rightarrow B \subseteq P(A)$. So let B be an arbitrary set with $B \subseteq A$. Then $(\forall x) (x \in B \rightarrow x \in A)$. We have to show that $(\forall x) [x \in B \rightarrow x \in P(A)]$. So let $x \in B$. Then $x \in A$ because $B \subseteq A$. But $A \subseteq P(A)$ Hence $x \in P(A)$. So if $x \in B$, then $x \in P(A)$ Hence we have shown that $(\forall x) [x \in B \rightarrow x \in P(A)]$ Since B was arbitrary we have shown that $(\forall B) [B \subseteq A \rightarrow B \subseteq P(A)]$. So $P(A) \subseteq P(P(A))$ provided $A \subseteq P(A)$.

Note: It is not always true that $A \subseteq P(A)$. That's why we need to assume it. [Take $A = \{1, 2\}$. Then $P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ so $A \notin P(A)$.]

Ch.3.3 #5 (a) Take $A = \emptyset$. Then $P(A) = \{\emptyset\}$ and $A \subseteq P(\emptyset)$

(i) because $\emptyset \subseteq \{\emptyset\}$

(b) Take $A = \{\emptyset, \{\emptyset\}\}$. Then

$$P(A) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} \text{ and again}$$

$$A \subseteq P(A) \text{ because } \{\emptyset, \{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}.$$

Note: It is true that $A \in P(A)$ for any set A
but that is a different story!

#6(a) Hint: Take $y = (1+2x)/(x-1)$.

(b) Suppose there is a real number y such that

$$(y+1)/(y-2) = x. \text{ Now, } \suppose x = 1. \text{ Then}$$

$$(y+1)/(y-2) = 1. \text{ So } y+1 = y-2$$

So $1 = -2$ — a contradiction.

Hence if there exists a real number y such that $(y+1)/(y-2) = x$, then $x \neq 1$.

#7. Hint: Take $y = (x + \sqrt{x^2-4})/2$ or $(x - \sqrt{x^2-4})/2$

(We get these values by solving $y^2 - xy + 1 = 0$)

#18(a) Suppose a/b and a/c . Then we can find integers k and l such that $b = k \cdot a$ and $c = l \cdot a$. So

$$b+c = k \cdot a + l \cdot a = (k+l) \cdot a. \therefore a/(b+c)$$

So if a/b and a/c , then $a/(b+c)$

(b) Suppose ac/bc and $c \neq 0$. Then we can find an integer k such that $bc = k \cdot (ac)$. Since $c \neq 0$, it follows that $b = k \cdot a$. Hence a/b .
So if ac/bc and $c \neq 0$, then a/b .

Ch. 3.3 #25 Hint: Take $y = 2x$.

Ch. 3.4 #1. Suppose $(\forall x)[P(x) \wedge Q(x)]$ is true. Let x be an arbitrary element. Then $P(x) \wedge Q(x)$ is true. So $P(x)$ is true and $Q(x)$ is true. Since x was arbitrary, $P(x)$ is true for all x . So $(\forall x)P(x)$ is true. Also since x was arbitrary $Q(x)$ is true for all x . So $(\forall x)Q(x)$ is true. Thus $(\forall x)P(x) \wedge (\forall x)Q(x)$ is true. Hence $(\forall x)[P(x) \wedge Q(x)]$ implies $(\forall x)P(x) \wedge (\forall x)Q(x)$.

Now prove $(\forall x)P(x) \wedge (\forall x)Q(x)$ implies $(\forall x)[P(x) \wedge Q(x)]$ and you will get the complete proof.

#2 Let $x \in A$. Since $A \subseteq B$, it follows that $x \in B$. Also since $A \subseteq C$, it follows that $x \in C$. So $x \in B$ and $x \in C$. Thus $x \in B \cap C$. So if $x \in A$, then $x \in B \cap C$. Hence $A \subseteq B \cap C$. So if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.

#3 We are given that $A \subseteq B$. Let C be any set. Take an arbitrary element x of C . Now suppose $x \in A$. Then $x \in B$ because $A \subseteq B$. So if $x \notin B$, then $x \notin A$ (contrapositive). Hence if $x \in C$ and $x \notin B$, then $x \in C$ and $x \notin A$. So $x \in C - B$ implies $x \in C - A$. Hence $C - B \subseteq C - A$. Since C was arbitrary, it follows that for every set C , $C - B \subseteq C - A$.

Ch. 3.6 #9(d) Suppose $A \Delta C = B \Delta C$. Then $(A - C) \cup (C - A) = (B - C) \cup (C - B)$.

Let $x \in A$. Then either $x \in C$ or $x \notin C$.

Now if $x \in C$, then $x \notin A - C$ and $x \notin C - A$. So

$x \notin (A - C) \cup (C - A)$. Hence $x \notin (B - C) \cup (C - B)$. So

$x \notin B - C$ and $x \notin C - B$. Since $x \in C$ and $x \notin C - B$, it follows that $x \in B$.

And if $x \notin C$, then $x \in A - C$. Since $A - C \subseteq (B - C) \cup (C - B)$, it follows that $x \in B - C$ or $x \in C - B$.

But $x \notin C$, so we must have $x \in B - C$. Hence $x \in B$.

So in either case $x \in B$. $\therefore A \subseteq B$.

Now let $x \in B$. Then either $x \in C$ or $x \notin C$ again.

If $x \in C$, then $x \notin (B - C)$ and $x \notin (C - B)$. So

$x \notin (B - C) \cup (C - B)$. Hence $x \notin (A - C) \cup (C - A)$. So

$x \notin A - C$ and $x \notin C - A$. Since $x \in C$ and $x \notin C - A$, it follows that $x \in A$.

And if $x \notin C$, then $x \in B - C$. Since $B - C \subseteq (A - C) \cup (C - A)$, it follows that $x \in (A - C)$ or $x \in (C - A)$.

But $x \notin C$, so we must have $x \in A - C$. Hence

$x \in A$. So in either case $x \in A$. $\therefore B \subseteq A$

Hence $A = B$. So if $A \Delta C = B \Delta C$, then $A = B$.

(This is proving 4(d) the hard way. Notice how much shorter, the previous proof of 4(d) was. We did need to use 4(a), 4(b) and the associative law for Δ , however. So it wasn't really that short.)

Ch.3.4 #4. Suppose $A \subseteq B$ and $A \not\subseteq C$. Then we can find an element $x_0 \in A$ such that $x_0 \notin C$ (because $A \not\subseteq C$). Now since $A \subseteq B$, $x_0 \in B$. So $x_0 \in B$ and $x_0 \notin C$. Hence $B \not\subseteq C$. So if $A \subseteq B$ and $A \not\subseteq C$, then $B \not\subseteq C$.

#7. Let $C \in P(A \cap B)$. Then $C \subseteq A \cap B$. So $C \subseteq A$ and $C \subseteq B$. Hence $C \in P(A)$ and $C \in P(B)$. Thus $C \in P(A) \cap P(B)$ - - - (1)

Now let $C \in P(A) \cap P(B)$. Then $C \in P(A)$ and $C \in P(B)$. So $C \subseteq A$ and $C \subseteq B$. Hence $C \subseteq A \cap B$. Thus $C \in P(A \cap B)$ - - - (2)

From (1) & (2) it follows that $P(A \cap B) = P(A) \cap P(B)$

#8. Suppose $A \subseteq B$. Let $C \in P(A)$. Then $C \subseteq A$. Since $A \subseteq B$, it follows that $C \subseteq B$. Thus $C \in P(B)$. So $P(A) \subseteq P(B)$. Thus $A \subseteq B$ implies $P(A) \subseteq P(B)$ - - - (1)

Now suppose $P(A) \subseteq P(B)$. Then $C \in P(A)$ implies $C \in P(B)$. Now we know that $A \in P(A)$ because $A \subseteq A$. So $A \in P(B)$, i.e. $A \subseteq B$.

Thus $P(A) \subseteq P(B)$ implies $A \subseteq B$. - - - (2)

From (1) & (2) it follows that $A \subseteq B$ iff $P(A) \subseteq P(B)$.

#9 Suppose x and y are odd. Then we can find integers k and l such that $x = 2k+1$ & $y = 2l+1$. So $xy = (2k+1)(2l+1) = 4kl + 2k + 2l + 1 = 2(2kl+k+l) + 1$ is odd. Hence if x & y are odd, then xy is odd

Ch. 3.4 #10. Suppose n is even. Then we can find an integer k such that $n = 2k$. So $n^3 = (2k)^3 = 2(4k^3)$ is even. So if n is even, then n^3 is even.

Now suppose n is not even. Then we can find an integer l such that $n = 2l+1$. So

$$n^3 = (2l+1)^3 = 8l^3 + 12l^2 + 6l + 1 = 2(4l^3 + 6l^2 + 3l) + 1$$

is odd. Hence if n is not even then n^3 is not even.

Thus n is even iff n^3 is even.

#12. Let $x \in \mathbb{R}$. Suppose $x \neq 1$. Take $y = x/(x-1)$. Then $y \in \mathbb{R}$

$$\begin{aligned} \text{Also } x+y &= x + \frac{x}{x-1} = \frac{x^2 - x}{x-1} + \frac{x}{x-1} \\ &= \frac{x^2}{x-1} = x \cdot \frac{x}{x-1} = xy \end{aligned}$$

So if $x \neq 1$, then $(\exists y \in \mathbb{R}) (x+y=xy)$.

Now suppose $(\exists y \in \mathbb{R}) (x+y=xy)$. If $x=1$, then we would get $1+y=1 \cdot y$, that is $1+y=y$. So $1=0$ - a contradiction.

Hence if $(\exists y \in \mathbb{R}) (x+y=xy)$ then $x \neq 1$.

Thus $(\forall x) [(\exists y \in \mathbb{R}) (x+y=xy) \Leftrightarrow x \neq 1]$

#26.(a) Suppose $3/n$ and $5/n$. Then $\exists k \in \mathbb{Z}$ such that $n = 3k$. Since $5/n$, $\exists l \in \mathbb{Z}$ such that $n = 5l$. Since $5 \nmid 3$, $5 \nmid k$. So $\exists m \in \mathbb{Z}$ such that $k = 5m$. Thus $n = 3k = 3(5m) = 15m$.

Hence $15/n$. Now suppose $15/n$. Then $\exists m \in \mathbb{Z}$ such that $n = 15m$. Since $n = 3(5m)$, $3/n$. Also since $n = 5(3m)$, $5/n$. Hence $15/n$ iff $3/n$ and $5/n$. (b)

A(21)

Ch.3.4 #26(b) $6/30$ and $10/30$ but $60 \not\propto 30$. So $6/n$ and $10/n$ does not imply $60/n$.

Ch.3.5 #1. Let $x \in A \cap (B \cup C)$. Then $x \in A$ and $x \in (B \cup C)$.

So $(x \in A) \wedge (x \in B \vee x \in C)$. Hence

$$(x \in A \wedge x \in B) \vee (x \in A \wedge x \in C) \quad (\text{Distributive Law})$$

So $(x \in A \cap B) \vee x \in (A \cap C)$. $\therefore x \in (A \cap B) \cup (A \cap C)$

$$\text{Hence } A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

2. Let $x \in (A \cup B) - C$. Then $x \in A - B$ and $x \notin C$. So

$$(x \in A \vee x \in B) \wedge (x \notin C)$$

$$(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C) \quad (\text{Distributive Law})$$

$\therefore (x \in A) \vee (x \in B \wedge x \notin C)$ because $x \in A \wedge x \notin C$

implies $x \in A$. Hence $x \in A$ or $x \in (B - C)$

$$\therefore x \in A \cup (B - C)$$

Thus $(A \cup B) - C \subseteq A \cup (B - C)$.

7. Let $C \in \mathcal{P}(A) \cup \mathcal{P}(B)$. Then $C \in \mathcal{P}(A)$ or $C \in \mathcal{P}(B)$

So $C \subseteq A$ or $C \subseteq B$. Now if $C \subseteq A$, then

$C \subseteq A \cup B$. And if $C \subseteq B$, then $C \subseteq A \cup B$. So

in either case $C \subseteq A \cup B$. $\therefore C \in \mathcal{P}(A \cup B)$

$$\text{Hence } \mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$$

10. Suppose $|x-3| > 3$. Now either $x-3 \geq 0$ or

$(x-3) < 0$. Call these Case (i) and Case (ii), resp.

Case (i): $x-3 \geq 0$. In this case $|x-3| = x-3$.

so we know that $x-3 > 3$. $\therefore x > 3+3=6$.

$$\therefore x^2 = x \cdot x > 6 \cdot x \text{ because } x > 6. \text{ So } x^2 > 6x$$

Ch. 3.5 #10. Case (ii): $x-3 < 0$. In this case $|x-3| = -(x-3)$.

So we know that $-(x-3) \geq 3$. $\therefore (-x)+3 > 3$

$\therefore -x > 0$. $\therefore x < 0$. So $6x < 0$, and

Since $x^2 > 0$ and $6x < 0$, it follows that $x^2 > 6x$.

So in either case $x^2 > 6x$. Hence if $|x-3| > 3$, then $x^2 > 6x$.

#13. Hint: $x^2 + x = x(x+1)$. Now observe that either x is even or $x+1$ is even.

#26 (a) The "proof" is not complete - so it is not correct.

(b) The "proof" was done by splitting the problem into two cases. We want to prove $P \rightarrow Q$. We split P into two cases $P_1 \vee P_2$.

(c) Yes, it can be fixed.

(d) The theorem is correct. We will fix the "proof." First, let us state what was wrong. In case 1 - we did not prove that $0 < x$. And in case 2 we did not prove that $x < 6$. Remember we have to prove $0 < x < 6$ in each case.

Case (1): $x-3 \geq 0$. Then ... so clearly $x < 6$. Also since $x-3 \geq 0$, $x \geq 3$. So $x > 0$. Hence $0 < x < 6$.

Case (2): $x-3 < 0$. Then ..., so $0 < x$. Also

since $x-3 < 0$, $x < 3$. So $x < 6$. Hence $0 < x < 6$. So in either case we got $0 < x < 6$. Hence the Theorem is true.

#31. This was already done in §3.3 #1

Ch.3.6 #1. (a) Let $y = x/(x^2+1)$. Then $y \in \mathbb{R}$ and

$$\begin{aligned} x - y &= x - \frac{x}{x^2+1} = \frac{x(x^2+1) - x}{x^2+1} \\ &= \frac{x^3}{x^2+1} = x^2 \cdot \frac{x}{x^2+1} = x^2 \cdot y \end{aligned}$$

So for every real number x , there exists a real number y such that $x^2y = x - y$.

(b) Suppose z is a real number such that $x^2z = x - z$. Then

$$x^2z + z = x. \quad \text{So } (x^2+1)z = x$$

$$\text{Hence } z = x/(x^2+1) = y.$$

So for every real number x , there exists a unique real number y such that $x^2y = x - y$.

#2.(a) Hint: Take $y = -1$. Then $xy + x - 4 = -x + x - 4 = -4 = 4y$.

(b) Hint: Assume $xz + x - 4 = 4z$. Then show that $z = -1$.

#9.(a) Hint: Take $X = \emptyset$. $(A - \emptyset) \cup (\emptyset - A) = A \cup \emptyset = A$

(b) Hint: let $B = A - A$. $A \Delta A = (A - A) \cup (A - A) = \emptyset$

(c) Hint: Take $C = A \Delta B$. Then:

$$A \Delta C = A \Delta (A \Delta B) = (A \Delta A) \Delta B \quad (\text{Assoc. Law})$$

$$= \emptyset \Delta B = B \Delta \emptyset = B \quad (\text{Comm. Law})$$

(d) Hint: Take $B = A$. Then for any $C \subseteq A$,

$$B \Delta C = A \Delta C = (A - C) \cup (C - A)$$

$$= (A - C) \cup \emptyset = A - C$$

$$(e) \quad A = A \Delta \emptyset = A \Delta (C \Delta C) = (A \Delta C) \Delta C \quad (\text{Assoc. Law})$$

$$= (B \Delta C) \Delta C = B \Delta (C \Delta C) = B \Delta \emptyset = B.$$

Ch. 3.7 #1 (Existence) Let $A = \cup \mathcal{I}$.

(a) Suppose $C \in \mathcal{I}$. Let $x \in C$. Then $x \in C$ and $C \in \mathcal{I}$. So $x \in \{x : (\exists D)(x \in D \text{ and } D \in \mathcal{I})\} = A$. Hence if $x \in C$, then $x \in A$. So $C \subseteq A$.

(b) Let B be any set such that $\mathcal{I} \subseteq P(B)$. Then $(\forall D)(D \in \mathcal{I} \rightarrow D \subseteq B)$. So for each $C \in \mathcal{I}$, $C \subseteq B$. Hence $\cup \{C : C \in \mathcal{I}\} \subseteq B$. But $A = \cup \{C : C \in \mathcal{I}\}$. Hence $A \subseteq B$.

(Uniqueness). Suppose A' was another set with the properties (a) and (b). Then $\mathcal{I} \subseteq P(A')$ and $(\forall B)(\mathcal{I} \subseteq P(B) \rightarrow A' \subseteq B)$. Now if we take $B = A$, then we know $\mathcal{I} \subseteq P(A)$, so it follows that $A' \subseteq A$.

Also if we take $B = A'$, and plug it into the original result that we proved for A , then we will get from $\mathcal{I} \subseteq P(A')$ and

$$(\forall B)(\mathcal{I} \subseteq P(B) \rightarrow A \subseteq B)$$

that $A \subseteq A'$. Hence $A = A'$.

So there is only one set A that satisfies (a) & (b).

$$\#2. P(A-B) - [P(A) - P(B)] = \{\emptyset\}. \text{ Let's prove this}$$

First observe that $P(A-B) \subseteq P(A)$ because

$$A-B \subseteq A. \text{ Also } P(A) - P(B) = P(A) - P(A \cap B).$$

$$\text{So } P(A-B) - [P(A) - P(B)] = P(A-B) \cap P(A \cap B).$$

Now $C \in P(A-B) \cap P(A \cap B)$, \therefore iff $C \subseteq A-B$

and $C \subseteq A \cap B$. The only set with this property

is \emptyset . So $C = \emptyset$. Thus $P(A-B) - [P(A) - P(B)] = \{\emptyset\}$.