3.14 Prove or disprove: Let G and H be two connected graphs of order n, where $V(G) = \{v_1, v_2, ..., v_n\}$. If there exists a one-to-one correspondence $\phi: V(G) \rightarrow V(H)$ such that

$$d_G(v_i,v_{i+1}) = d_H(\phi(v_i),\phi(v_{i+1}))$$

for all i $(1 \le i \le n - 1)$, then $G \cong H$.

Solution: This is easily seen to be false. Let $G = P_n$, and let $H = C_n$ for $n \ge 3$. We may realize these two graphs as follows:

Set

$$V(G) = V(H) = \{v_1, v_2, ..., v_n\},$$

and then let

$$E(G) = \{ v_iv_{i+1} : 1 \le i \le n - 1 \},$$

and

$$E(H) = \{ v_iv_{i+1} : 1 \le i \le n - 1 \} \cup \{ v_nv_1 \}.$$

We define $\phi: V(G) \rightarrow V(H)$ by $\phi(v_i) = v_i$ for $i = 1, ... , n$.

Plainly, we have

$$d_G(v_i,v_{i+1}) = 1 = d_H(\phi(v_i),\phi(v_{i+1}))$$

for all i $(1 \le i \le n - 1)$, but the two graphs are not isomorphic, since all of the vertices of H are of degree 2, but not all those of G are.//