

4.8 Prove that if every vertex of a graph G has degree at least 2, then G contains a cycle.

Proof: We shall prove the contrapositive, i.e., if G contains no cycles, then G has a vertex with degree less than 2. To this end, suppose that G has no cycles. Then G must be a forest. Let T be a component of G . If T is trivial, then T , and thus G , has a vertex of degree 0. If T is nontrivial, then T is a nontrivial tree and Theorem 4.3 implies that T has at least two end-vertices. These are of degree 1. Consequently, if G has no cycles, then G has at least one vertex with degree less than 2.//