9.8 Determine, with explanation, whether the graph $K_4 \times K_2$ is planar.

Solution:
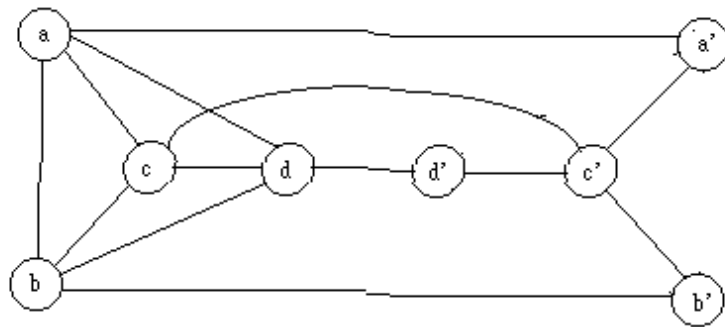
$$K_4 \times K_2$$

a   a'
c   d   d'   c'
b   b'
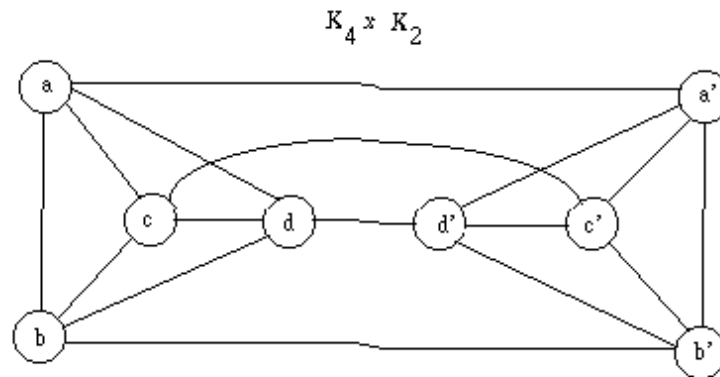
Here is a subdivision of

$$K_5$$

in

$$K_4 \times K_2 .$$

$$V(K_5) = \{\, a, b, c, d, c' \,\}.$$

a   a'
c   d   d'   c'
b   b'

Here a couple of pictures are worth a vexation of verbosity. The crux of the matter is that since $K_4 \times K_2$ contains a subgraph that is isomorphic to a subdivision of K_5, Kuratowski's Theorem implies that $K_4 \times K_2$ is not planar. In the first diagram, above, we have a realization of the graph $K_4 \times K_2$. Notice that it appears that at least one crossing of edges seems necessary.

The second diagram, above, reveals the isomorph of the subdivision of K_5.

[The pain here was in ruling out a subdivision of $K_{3,3}$ as a candidate. Of course I may be mistaken.]