30. Give an example of a partially ordered set that has a unique minimal element but no smallest element.

Construction: Set $A = \{ (x,y) : x=2 \text{ and } y=1, \text{ or } x=1 \text{ and } y \in \mathbb{N} \}$,

and define a partial order on A, denoted by <, as follows:

$$(x_1,y_1) < (x_2,y_2)$$

when $x_1 > x_2$ and $y_1 = y_2 = 1$,

or

when $x_1 = x_2 = 1$ and $y_1 > y_2$.

Here, of course, the order between component elements is the natural order defined on the natural numbers.

It is both routine and tedious to verify that we have defined a partial order on the set A. The order is not a simple order since (2,1) < (1,1) and (2,1) relates to none of the other elements in A. Evidently, for each $n \in \mathbb{N}$, (1,n) > (1,n+1). Consequently, none of the elements of the form (1,n) is minimal. Thus, we have a unique element, (2,1), that is minimal, but not a least element. Here is a lattice diagram of the situation:

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          (1,1)
         /     \
     (2,1)     (1,2)
                 \
                 (1,3)
                   \
                    ⋱
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