3-12. Let <E_n> be any sequence of disjoint measurable sets and A any set. Then

$$m^*(A \cap \bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} m^*(A \cap E_i).$$

Proof. From Proposition 3-2, which deals with the countable subadditivity of outer measure, we have

$$m^*(A \cap \bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^*(A \cap E_i).$$

From the monotonicity of outer measure and Lemma 3-9, we have

$$m^*(A \cap \bigcup_{i=1}^{\infty} E_i) \geq m^*(A \cap \bigcup_{i=1}^{n} E_i)$$

$$= \sum_{i=1}^{n} m^*(A \cap E_i)$$

for each $n \varepsilon \mathbb{N}$. Since the sum of the series is, in fact, the supremum of the set of partial sums, we are done.//

3-15. Show that if E is measurable and $E \subset P$, then $m(E) = 0$.

Proof. Let E be a measurable subset of P, defined on page 65, and nonmeasurable. Let $E_i = E \oplus r_i$, where then operation here is addition modulo one on the interval [0,1) and <r_i> is an enumeration of the rational numbers in [0,1) with $r_0 = 0$, as in the definition of the set P. Evidently, the sequence of sets is pairwise disjoint since $E_i \subset P_i$ for each i in the nonnegative integers. From Lemma 3-16, $m(E_i) = m(E)$ for each i. Since we have $\cup E_i \subset [0,1)$, $\sum m(E_i) = m(\cup E_i) \leq 1$. Consequently, $m(E_i) = 0$ for every i, since the sum is not finite. Thus, $m(E) = 0$.//