
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me the magic on the page.*

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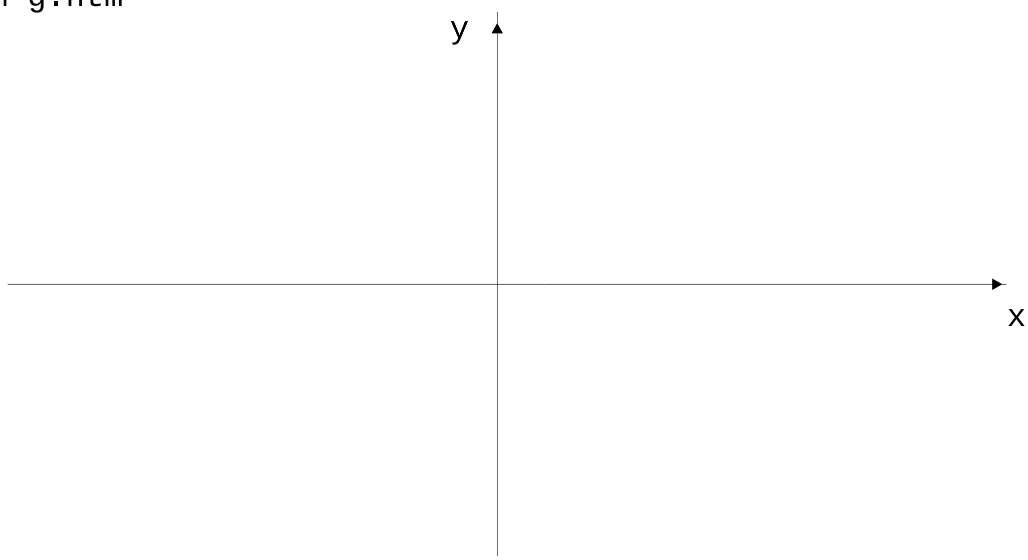
1. (10 pts.) On the coordinate system below sketch the graph of the function g defined below. Label very carefully.

$$g(x) = \begin{cases} (x - 1)^2 & , \quad x \geq 1 \\ 1 - x & , \quad x < 1 \end{cases}$$

See c1-t1m-g.htm



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2. (10 pts.) If $f(x) = 3x^2 - x$, find

$$\frac{f(x + h) - f(x)}{h} ,$$

and simplify as much as possible algebraically. Kindly observe that there are no limits of any sort being taken here.

$$\frac{f(x + h) - f(x)}{h} = \frac{[3(x+h)^2-(x+h)] - [3x^2-x]}{h}$$

$$= 6x - 1 + 3h \quad \text{for } h \neq 0$$

after the algebraic dust settles.

3. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definition of a **function** f //

A function f is a rule that associates a unique output with each input. If the input is denoted by x, then the output is denoted by f(x). [Note: The uniqueness of the output is critical.]

4. (10 pts.) Suppose $f(x) = x^2 - 3x$ and $g(x) = (x - 4)^{1/2}$. Obtain a formula for each of the following functions and state clearly what their domains are:

(a) $(f + g)(x) = x^2 - 3x + (x - 4)^{1/2}$

$\mathrm{dom}(f + g) = [4,\infty)$

(b) $(f\cdot g)(x) = (x^2 - 3x)(x - 4)^{1/2}$

$\mathrm{dom}(f\cdot g) = [4,\infty)$

(c) $(f/g)(x) = (x^2 - 3x)/(x - 4)^{1/2}$

$\mathrm{dom}(f/g) = (4,\infty)$

(d) $(f - g)(x) = x^2 - 3x - (x - 4)^{1/2}$

$\mathrm{dom}(f - g) = [4,\infty)$

(e) $(g\circ f) = g(f(x))$

$= ((x^2 - 3x) - 4)^{1/2} = (x^2 - 3x - 4)^{1/2}$

$\mathrm{dom}(g\circ f) = \{ x \mid f(x) \varepsilon \mathrm{dom}(g) \}$

$= \{ x \mid x^2 - 3x \geq 4 \}$

$= \{ x \mid (x - 4)(x + 1) \geq 0 \}$

$= (-\infty,-1] \cup [4,+\infty)$

5. (5 pts.) Express $f(x) = 2\cdot\tan^2(3x^3)$ as the composition of two functions g and h with $f = g\circ h$, that is find g and h so that $f(x) = (g\circ h)(x)$. There are actually infinitely many correct answers to this. Here is, perhaps, one of the most obvious.

$g(x) = 2\cdot\tan^2(x)$

$h(x) = 3x^3$

Now check your work by correctly computing $(g\circ h)(x)$. [DETAILS REQUIRED!!] $(g\circ h)(x) = g(h(x)) = 2\cdot\tan^2(h(x)) = 2\cdot\tan^2(3x^3)$

6. (20 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of page two when you run out of room here.]

(a) $$\lim_{x \to +2} \frac{x + 2}{x^2 - 4} = \lim_{x \to +2} \frac{1}{x - 2}$$ fails to exist.

(b) $$\lim_{x \to -2} \frac{x + 2}{x^2 - 4} = \lim_{x \to -2} \frac{1}{x - 2} = -1/4$$

(c) $$\lim_{t \to +\infty} \frac{6 - 14t^3}{2t^2 + 3} = \lim_{t \to +\infty} \frac{t^3[6t^{-3} - 14]}{t^2[2 + 3t^{-2}]} = -\infty$$

(d) $$\lim_{x \to +\infty} \frac{7x^4 - 3x^3}{x + 3x^4} = \lim_{x \to +\infty} \frac{7 - 3x^{-1}}{x^{-3} + 3} = 7/3$$

7. (10 pts.) Suppose that

$$h(x) = \begin{cases} x^2 - 3x + 7 & \text{, if } x < 1 \\ 10 & \text{, if } x = 1 \\ 5x & \text{, if } x > 1 \end{cases}$$

Evaluate the following limits:

(a) $\lim_{x \to 1^+} h(x) = \lim_{x \to 1^+} 5x = 5$

(b) $\lim_{x \to 1^-} h(x) = \lim_{x \to 1^-} (x^2 - 3x + 7) = 5$

(c) What can you conclude from parts (a) and (b)? Why??

Since the left and right limits are the same, $\lim_{x \to 1} h(x) = 5$.

[You might observe that the function h fails to be continuous at x = 1. The important thing at this stage is that you recognize that the limit exists since the two one-sided limits exist and have the same value.]

8. (10 pts.) Evaluate each of the following thorny limits:

(a) $\lim_{x \to \infty} ((x^2 + 36x)^{1/2} - x) = \lim_{x \to \infty} \dfrac{36x}{(x^2 + 36x)^{1/2} + x}$

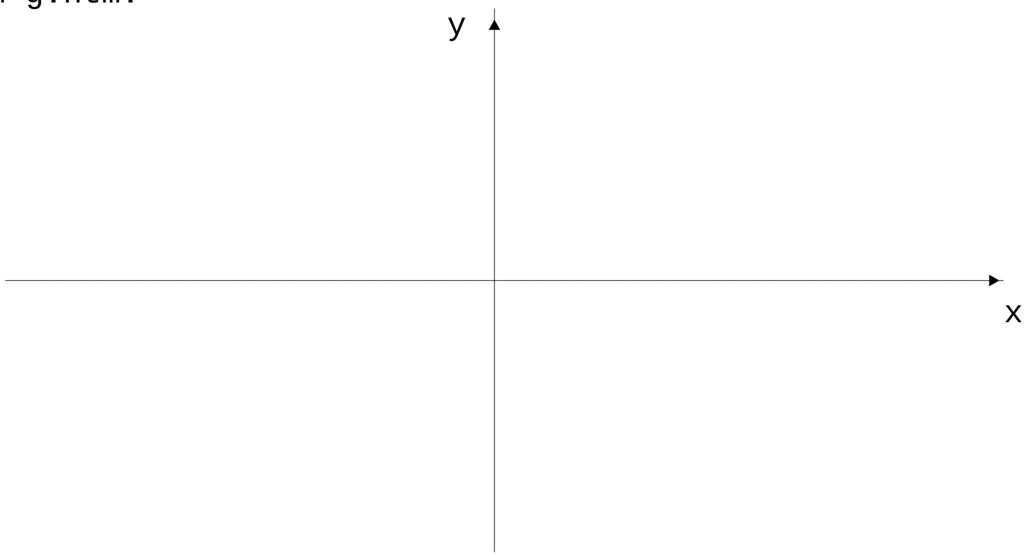
$= \lim_{x \to \infty} \dfrac{36x}{|x|(1 + 36x^{-1})^{1/2} + x} = 18$

(b) $\lim_{x \to 0} \dfrac{(x + 36)^{1/2} - 6}{x} = \lim_{x \to 0} \dfrac{1}{(x + 36)^{1/2} + 6} = 1/12$

The key piece of algebraic magic above is captured in the prestidigitation of rationalizing the numerator. Why would one dream of doing that?? Look at the form ... .

9. (10 pts.) Sketch the curve defined by the parametric equations $x = 1 + \cos(t)$ and $y = -1 - \sin(t)$ with $0 \le t \le 2\pi$ by eliminating the parameter, and indicate the direction of increasing t. // Mr. Pythagoras allows us to eliminate the parameter t and write $(x - 1)^2 + (y + 1)^2 = 1$. This wheelie is done in the negative sense.

See c1-t1m-g.htm.



10. (5 pts.) Express the following function in piecewise defined form without using absolute values:

$$f(x) = 8x - |x - 1|$$

$$f(x) = \begin{cases} 8x - (x - 1) & , \text{ if } x \ge 1 \\ 8x - [-(x - 1)] & , \text{ if } x < 1 \end{cases}$$

$$= \begin{cases} 7x + 1 & , \text{ if } x \ge 1 \\ 9x - 1 & , \text{ if } x < 1 \end{cases}$$

11. (5 pts.) Given $\lim_{x\to a} [3f(x) - 2g(x)] = -2$ and $\lim_{x\to a} g(x) = -4$,

$$\lim_{x\to a} f(x) = (1/3)\lim_{x\to a}\{[3f(x) - 2g(x)] + 2g(x)\}$$

$$= (1/3)(-2 + (-8)) = -10/3$$

$$\lim_{x\to a} [g(x)]^3 = [-4]^3 = -64$$