Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

1. (10 pts.) (a) Using implicit differentiation, compute dy/dx and d^2y/dx^2 when $x^2 + y^2 = 25$. **Label your expressions correctly or else.**

(b) Obtain an equation for the line tangent to the graph of $x^2 + y^2 = 25$ at the point $(1,-(24)^{1/2})$.

2. (5 pts.) A 5-ft. ladder is leaning against the wall. If the top of the ladder slips down the wall at a rate of 2 ft./sec., how fast will the foot be moving away from the wall when the top is 3 ft. above the ground?

3. (5 pts.) Use logarithmic differentiation to find dy/dx when $y = x^{\sin(x)}$. **Label your expressions correctly or else.**

4. (15 pts.) Differentiate the following functions. Do not attempt to simplify the algebra.

(a) $f(x) = \ln(3x^3 - x) - 2\cdot\exp(4x^3 - 14)$

$f'(x) =$

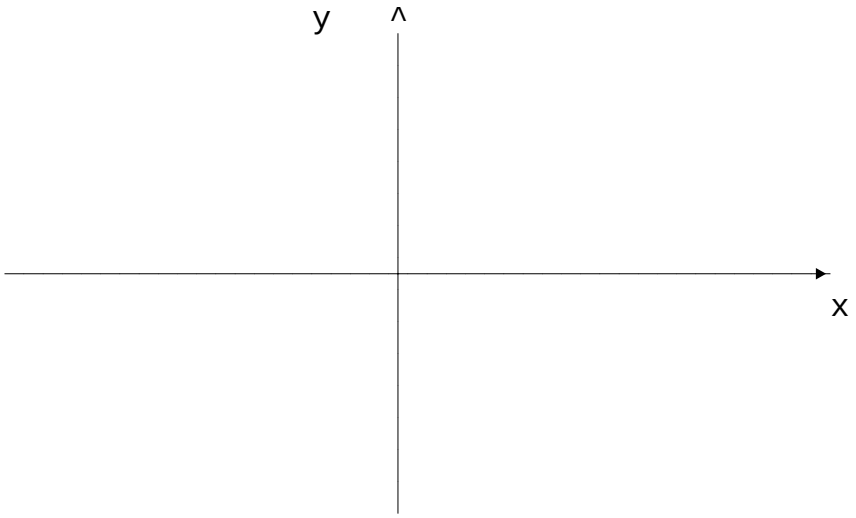
(b) $g(x) = 4^x + x^4 + 4^4 + \log_4(x) + \ln(4)$

$g'(x) =$

(c) $h(x) = \sec^{-1}(3x) + e^x\cdot\tan^{-1}(x) - 6\cdot\cos^{-1}(x^2)$

$h'(x) =$

5. (5 pts.) Carefully sketch the graph of $y = \tan^{-1}(x)$. Label very carefully.

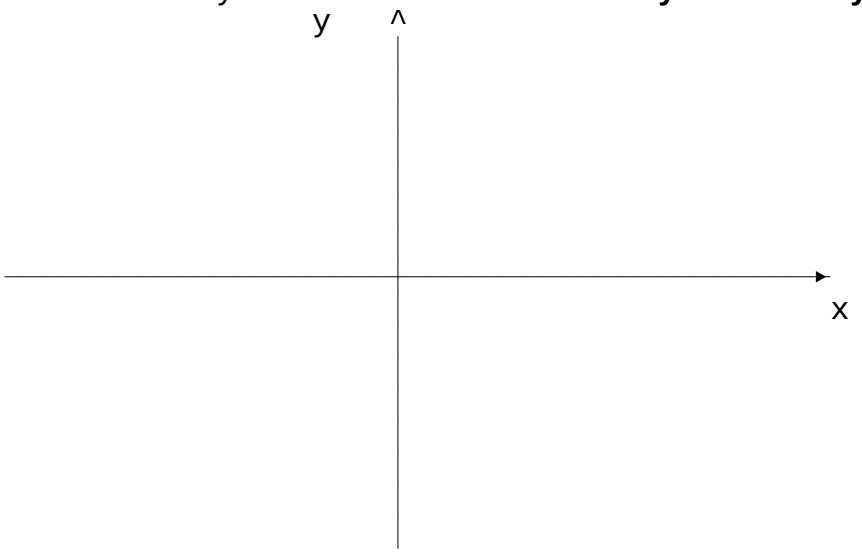


6. (5 pts.) Solve for x without using a calculator. Use the natural logarithm when logarithms are needed.

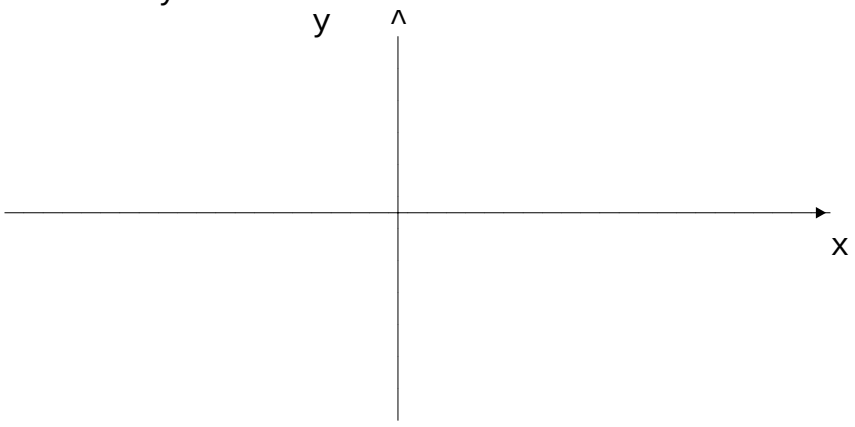
$e^{2x} - 6e^{x} = -8$

7. (5 pts.) Using a complete sentence and appropriate notation, provide the precise mathematical definitions for the following term: // The differential, dy, of a function f(x) //

8. (5 pts.) Carefully sketch both $f(x) = \ln(x)$ and $g(x) = e^{x}$ on the coordinate system below. **Label very carefully.**



9. (5 pts.) Carefully sketch the graph of $y = \sin^{-1}(x)$. Label very carefully.



10. (10 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\lim_{t \to \infty} 6t \cdot \tan(3 \cdot t^{-1}) =$

(b) $\lim_{x \to 0} \dfrac{1 - \cos(3\pi x)}{e^x + e^{-x} - 2} =$

11. (5 pts.) The side of a cube is measured to be 30 feet with a possible error of ±0.5 feet. Use differentials to estimate the relative error in the computed volume.

12. (5 pts.) Find the exact value of $\cos[2 \cdot \cos^{-1}(3/5)]$. [**Warning**: You will have to use some identities to handle this.]

13. (5 pts.) Let $f(x) = 4x^3 + 5x$. (a) Show f is invertible. (b) Then solve the equation $f^{-1}(x) = -2$.

14. (5 pts.) Use differentials and a linear approximation formula to estimate $(24)^{1/2}$. [Hint: Use $x_0 = 25$ and $f(x) = x^{1/2}$.]

15. (5 pts.) Solve for x without using a calculator.

$$\ln(16x) - 2\cdot\ln(x^2) = \ln(2)$$

16. (5 pts.) Carefully sketch the graph of $y = \cos^{-1}(x)$. Label very carefully.