**Student Number:** **Exam Number:**

**Read Me First:** *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

1. (30 pts.) Very basic derivatives .... Provide the first derivative for each of the following functions.

(a) $f(x) = x^{23}$ $f'(x) =$

(b) $f(x) = 23^{x}$ $f'(x) =$

(c) $f(x) = \ln(x)$ $f'(x) =$

(d) $f(x) = e^{x}$ $f'(x) =$

(e) $f(x) = \sin(x)$ $f'(x) =$

(f) $f(x) = \cos(x)$ $f'(x) =$

(g) $f(x) = \tan(x)$ $f'(x) =$

(h) $f(x) = \sec(x)$ $f'(x) =$

(i) $f(x) = \csc(x)$ $f'(x) =$

(j) $f(x) = \cot(x)$ $f'(x) =$

(k) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(l) $f(x) = \tan^{-1}(x)$ $f'(x) =$

(m) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(n) $f(x) = \cos^{-1}(0)$ $f'(x) =$

(o) $f(x) = \log_{23}(x)$ $f'(x) =$

2. (30 pts.) Very basic antiderivatives .... Give each of the following antiderivatives. Do not forget the arbitrary constant.

(a) $\int e^x \, dx$ =

(b) $\int x^{23} \, dx$ =

(c) $\int \sin(x) \, dx$ =

(d) $\int x^{-1} \, dx$ =

(e) $\int \cos(x) \, dx$ =

(f) $\int \frac{1}{1 + x^2} \, dx$ =

(g) $\int \frac{1}{(1 - x^2)^{1/2}} \, dx$ =

(h) $\int \sec(x) \cdot \tan(x) \, dx$ =

(i) $\int \sec^2(x) \, dx$ =

(j) $\int \frac{1}{|x|(x^2 - 1)^{1/2}} \, dx$ =

(k) $\int \csc(x) \cdot \cot(x) \, dx$ =

(l) $\int \csc^2(x) \, dx$ =

(m) $\int 0 \, dx$ =

(n) $\int 23^x \, dx$ =

(o) $\int 23^{23} \, dx$ =

3. (20 pts.) Some basic limits .... For each of the following, find the limit if it exists. If the limit does not exist, say so. Be as precise as possible in doing this. [Warning: At least one of these involves the definition of the derivative!!]

(a) $\displaystyle \lim_{x \to -5^+} \frac{x + 5}{x^2 - 25} =$

(b) $\displaystyle \lim_{x \to +\infty} \frac{4 - 9x^4 + 3x^2}{5x^4 - 11x + 11} =$

(c) $\displaystyle \lim_{\theta \to 0} \frac{\tan(9\pi\theta)}{\sin(3\pi^2\theta)} =$

(d) $\displaystyle \lim_{\theta \to 0} \frac{\sin(3\pi + \theta) - \sin(3\pi)}{\theta} =$

(e) $\displaystyle \lim_{x \to -\infty} 2 \cdot \tan^{-1}(x) =$

4. (20 pts.) More devious derivatives to do ... .
Compute the first derivative of each of the following functions.
Do not simplify the algebra.

(a) $f(x) = 5\cdot x^4 - 20\cdot \cos(x) + 30\cdot \csc^{-1}(x) + \ln(x^4) - 8\cdot e^{\tan(\pi/4)}$

$f'(x) =$

(b) $y = \sec(3x^2 - 4)\cdot e^{x^3}$

$\frac{dy}{dx} =$

(c) $g(\theta) = \dfrac{\tan(\theta) - 4\cdot \sin(\theta)}{\cot(\theta)}$

$\frac{dg(\theta)}{d\theta} =$

(d) $h(t) = \log_2(3t) - 4^{3t} + (4\cdot t^3)^2 + \ln(23)$

$h'(t) =$

5. (25 pts.) More ignominious integrals .... Evaluate each of the following antiderivatives or indefinite integrals.

(a) $$\int 6x^5 - \frac{9}{x} - 5\cdot\cos(x)\ dx =$$

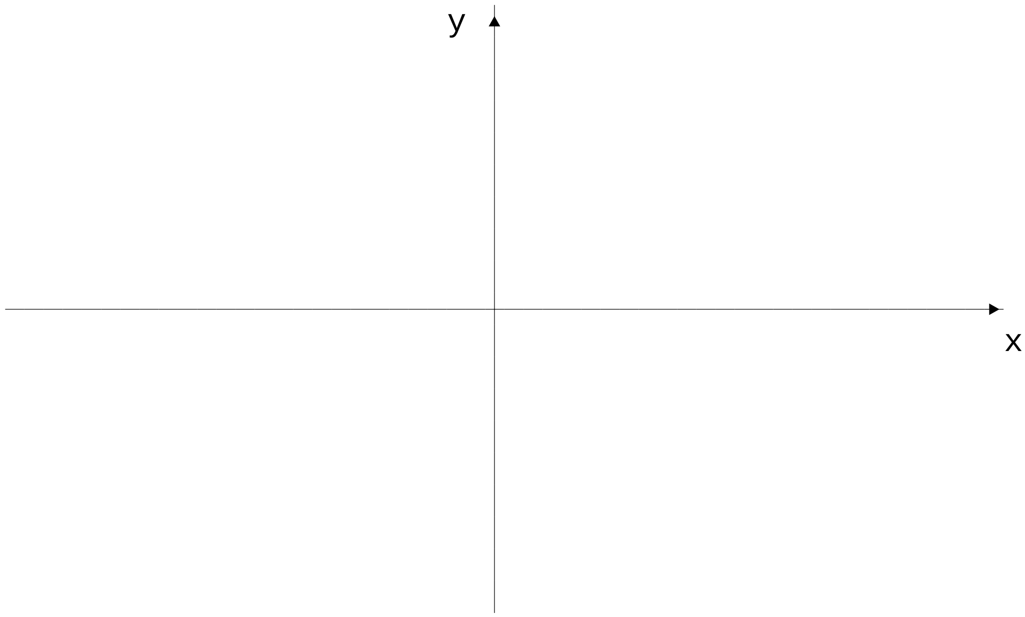
(b) $$\int e^{3x + 9} - \frac{20\cdot x}{x^2 + 1} + \frac{20}{x^2 + 1}\ dx =$$

(c) $$\int 4\cdot\cos^3(\theta)\cdot\sin(\theta)\ d\theta =$$

(d) $$\int (12t + 42)\cdot(t^2 + 7t)\ dt =$$

(e) $$\int \sin^3(x)\ dx =$$

6. (10 pts.) Let $f(x) = x^3 - x^4$ . Provide a complete analysis of this function on the back of Page 5. Then, below, plot the zero(s), the critical point(s), and the inflection point(s) of f. Finally, connect the dots and display the behavior of f at $\pm\infty$. Label very carefully.




---

7. (10 pts.) A person 6 ft. tall is walking at a rate of 3 ft./sec. toward a streetlight 18 feet high.

(a) At what rate is his shadow length changing?

(b) How fast is the tip of his shadow moving??

[Hints: (1) Draw a picture. (2) Look for similar triangles.]

---

8. (5 pts.) Give an equation for the line tangent to the graph of $f(x) = \tan(3x)$ at $x = \pi/12$.

9. (5 pts.) Use the Mean-Value Theorem to show that if f is continuous on the closed interval [a,b] and f'(x) = 0 for every x in the open interval (a,b), then f(x) = f(a) for every x in the closed interval [a,b].

10. (10 pts.) Provide a complete evaluation each of the following limits. In particular, point out any use of L'Hopital's Rule, and when *squeezing*, give a complete argument.

(a) $\lim_{t \to \infty} 16t^{-2} \cdot \sin(\pi \cdot t) =$

(b) $\lim_{x \to 0} \dfrac{2e^{2x} + 2e^{-2x} - 4}{1 - \cos(3x)} =$

11. (10 pts.) A rectangular area of 3200 square ft. is to be fenced off. It turns out that two opposite sides will use fencing costing $1 per foot, and the remaining two sides require fencing costing $2 per foot. Find the dimensions of the rectangle that requires the least cost. [In doing this, you must provide a sufficiently complete argument to convince the skeptics that your relative extremum is absolute!!]

12. (5 pts.) Solve the following initial value problem:
$y'(x) = 4 \cdot \cos(x) + 4$ and $y(\pi/2) = 8$.

13. (10 pts.) Using implicit differentiation, compute $d^2y/dx^2$ when $x^3 + y^3 = 1$.

14. (5 pts.) Use logarithmic differentiation to find dy/dx when

$$y = (30^{\tan(x)})/(5^{\ln(27\cos(x))}).$$

15. (5 pts.) Give a complete $\varepsilon$ - $\delta$ proof that

$$\lim_{x \to 1} (16x - 4) = 12.$$