Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful.   Remember this: "=" denotes "equals" , "$\Rightarrow$" denotes "implies" , and "$\Leftrightarrow$" denotes "is equivalent to".   Do not "box" your answers. Communicate. Show me all the magic on the page.*

1. (10 pts.)   (a) Using implicit differentiation, compute dy/dx and d^2y/dx^2 when $x^2 + y^2 = 16$. **Label your expressions correctly or else.**

(b) Obtain an equation for the line tangent to the graph of $x^2 + y^2 = 16$ at the point $(1,-(15)^{1/2})$.

2. (5 pts.)   A 5-ft. ladder is leaning against the wall. If the top of the ladder slips down the wall at a rate of 2 ft./sec., how fast will the foot be moving away from the wall when the top is 4 ft. above the ground?

3. (5 pts.) Use logarithmic differentiation to find dy/dx when $y = x^{\cos(x)}$. **Label your expressions correctly or else.**

4. (15 pts.)   Differentiate the following functions. Do not attempt to simplify the algebra.

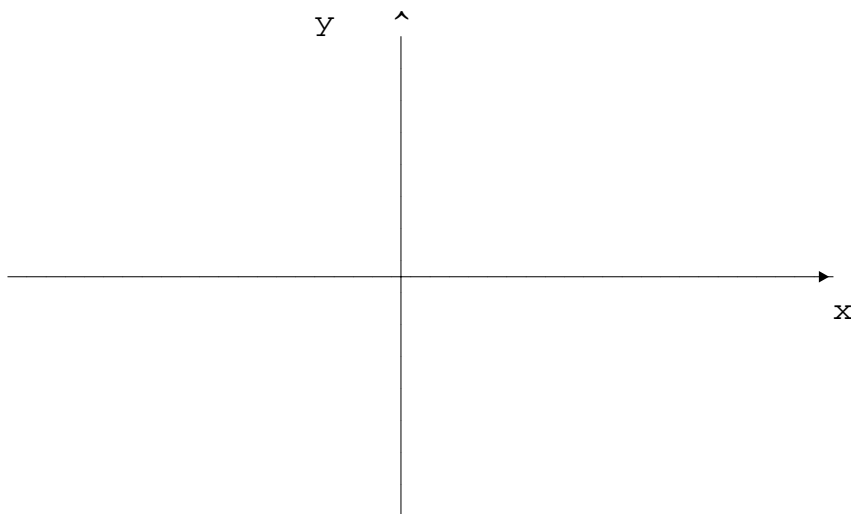(a) $f(x) = \ln(3x^3 - 7x) - 2 \cdot \exp(5x^3 - 14)$

     $f'(x) =$

(b) $g(x) = 5^x + x^5 + 5^5 + \log_5(x) + \ln(5)$

     $g'(x) =$

(c) $h(x) = \sec^{-1}(4x) + e^x \cdot \tan^{-1}(x) - 8 \cdot \cos^{-1}(x^2)$

     $h'(x) =$

5. (5 pts.) Carefully sketch the graph of $y = \tan^{-1}(x)$. Label very carefully.

6. (5 pts.)   Solve for x without using a calculator. Use the natural logarithm when logarithms are needed.
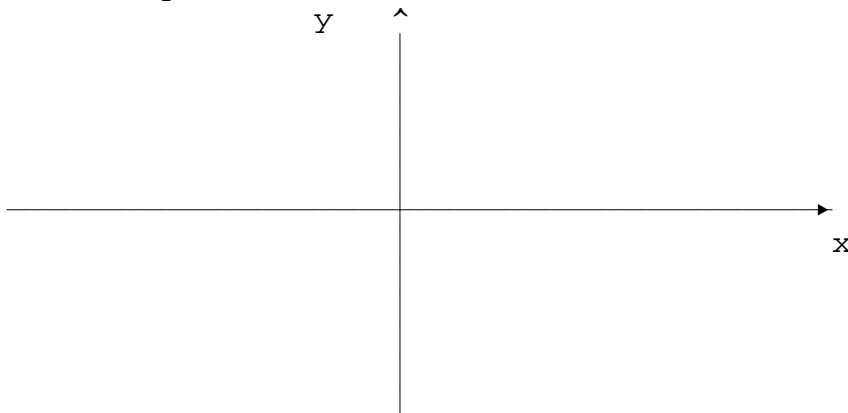
$$e^{2x} - 8e^x = -15$$

7. (5 pts.) Using a complete sentence and appropriate notation, provide the precise mathematical definitions for the following term: // The differential, dy, of a function f(x) //

8. (5 pts.) Carefully sketch both $f(x) = \ln(x)$ and $g(x) = e^x$ on the coordinate system below. **Label very carefully**.

Y   ^

|

————————————+————————————>
                         x
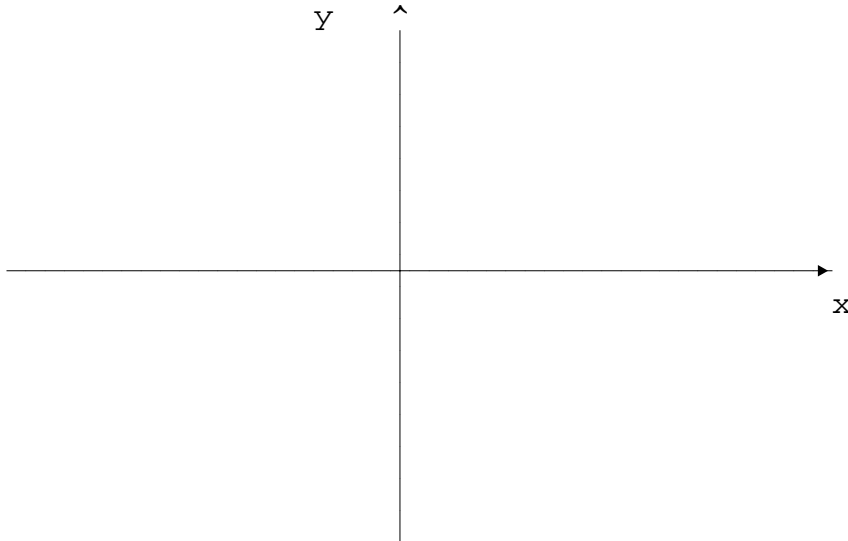
9. (5 pts.) Carefully sketch the graph of $y = \sin^{-1}(x)$. Label very carefully.

Y   ^

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————————————+————————————>
                         x

10. (10 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\lim\limits_{t \to \infty} 7t \cdot \tan(3 \cdot t^{-1})$ =

(b) $\lim\limits_{x \to 0} \dfrac{1 - \cos(5\pi x)}{e^x + e^{-x} - 2}$ =

11. (5 pts.) The side of a cube is measured to be 15 feet with a possible error of ±0.5 feet. Use differentials to estimate the relative error in the computed volume.

12. (5 pts.) Find the exact value of $\cos[2 \cdot \sin^{-1}(3/5)]$. [**Warning**: You will have to use some identities to handle this.]

13. (5 pts.) Let $f(x) = 5x^3 + 5x$. (a) Show f is invertible.
(b) Then solve the equation $f^{-1}(x) = -2$.

14. (5 pts.) Use differentials and a linear approximation
formula to estimate $(15)^{1/2}$. [Hint: Use $x_0 = 16$ and $f(x) = x^{1/2}$.]

15. (5 pts.)   Solve for x without using a calculator.

$\ln(81x) - 2 \cdot \ln(x^2) = \ln(3)$

16. (5 pts.) Carefully sketch the graph of $y = \cos^{-1}(x)$.   Label
very carefully.

y

x