**Read Me First:** *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

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1. (16 pts.) Fill in the blanks of the following analysis with the correct terminology.

Let $f(x) = x^4 - 8x^3$. Then $f'(x) = 4x^3 - 24x^2 = 4(x - 0)^2(x - 6)$. Consequently, $x = 0$ and $x = 6$ are ____________________ points of f. Since $f'(x) > 0$ for $6 < x$, f is ____________________ on the set $(6,\infty)$. Also, because $f'(x) < 0$ when $0 < x < 6$ or $x < 0$, and f is continuous, f is ____________________ on the interval $(-\infty , 6)$. Using the first derivative test, it follows that f has a(n) ______________________________________ at $x = 6$, and ______________________________________ at $x = 0$.

Since $f''(x) = 12x^2 - 48x = 12x\cdot(x - 4)$, we have $f''(0) = 0$, $f''(4) = 0$, $f''(x) < 0$ when $0 < x < 4$, and $f''(x) > 0$ when $x > 4$ or $x < 0$. Thus, f is __________________________ on the interval $(0,4)$, f is ________________________ on the set $(-\infty ,0) \cup (4,\infty)$, and f has _____________________________ at $x = 0$ and $x = 4$.

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2. (4 pts.) Rolle's Theorem states that if f(x) is continuous on [a,b] with f(a) = f(b) = 0 and differentiable on (a,b), then there is a number c in (a,b) such that $f'(c) = 0$. **Give an example of a function f(x) defined on [-1,1] with f differentiable on (-1,1) and f(-1) = f(1) = 0 but such that there is no number c in (-1,1) with f′(c) = 0.** [Hint: Which hypothesis above must you violate??]

3. (10 pts.) Find all the critical points of the function $f(x) = 3\cdot(x^2 + 8x)^{1/3}$. Which critical points are stationary points? Apply the second derivative test at each stationary point and draw an appropriate conclusion.

4. (10 pts.) Locate and determine the maximum and minimum values of the function $f(x) = 2x^3 + 6x^2$ on the interval [-4,-1]. What magic theorem allows you to conclude that f(x) has a maximum and minimum even before you attempt to locate them? Why??

5. (10 pts.) (a) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) Show how to use the Mean Value Theorem to prove the following: If x and y are real numbers, then

(*) $|\sin(2x) - \sin(2y)| \leq 2|x - y|$

is true. [Hint: If $x = y$ , life is easy. Why?? So pretend $x \neq y$. Without any loss of generality, $x < y$. Study $f(t) = \sin(2t)$ on the interval $[x,y]$.]

6. (10 pts.) A small rectangular area of 36 square feet is to be fenced. Three of the sides will use fencing costing \$2.00 per running foot, and the remaining side will use a hedge costing \$1.00 per running foot. Find the dimensions of the rectangle which has the least cost to enclose. Provide a complete enough analysis to convince the doubtful that your extreme value is an absolute minimum.

7. (5 pts.) Find the function f(x) that satisfies the following two equations: $f'(x) = \cos(x) - \sin(x)$ for all x and $f(\pi/2) = 4$.

8. (5 pts.) Find a function g(x) so that g satisfies the following equation:

$$\int g(x)dx = \sec^2(x) - 12x + 2e^x + C$$

g(x) =

9. (10 pts.) Evaluate the following antiderivatives.

$$\int 6x^5 - \frac{7}{x} - \frac{12}{x^2} - \frac{11}{|x|(x^2 - 1)^{1/2}}\, dx =$$

$$\int e^{2x - 3} - \frac{10 \cdot \sin(x)}{\cos^2(x) + 1}\, dx =$$

10. (20 pts.) Very carefully sketch each of the following functions. Label very carefully.

(a) $f(x)$ is continuous on $\mathbb{R}$ and satisfies the following:

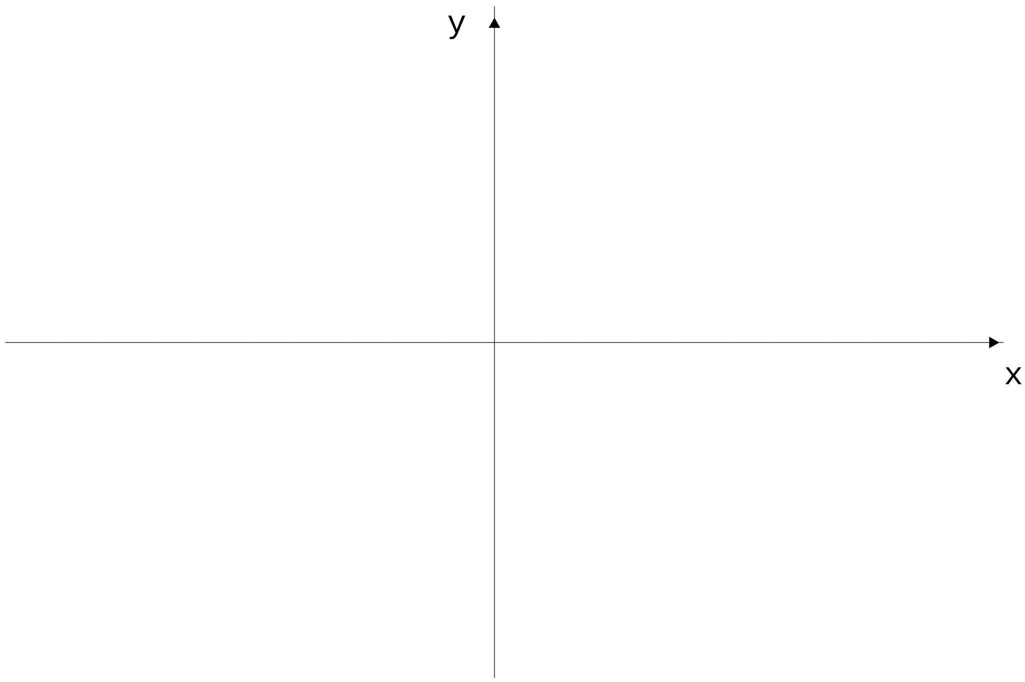
(1) $f(0) = 0$;

(2) $x < 0 \Rightarrow f'(x) > 0$, and $x > 0 \Rightarrow f'(x) < 0$;

(3) $\lim_{x \to 0^-} f'(x) = +\infty$ and $\lim_{x \to 0^+} f'(x) = -\infty$;

(4) $x \neq 0 \Rightarrow f''(x) > 0$; and

(5) $f(x) \longrightarrow -2$ as $x \longrightarrow \pm\infty$.




(b) $g(x) = 2x/(x^2 + 4)$ [Analyze $g'$ and $g''$ and how $g$ behaves as $x \longrightarrow \pm\infty$. Work on the back of page 4!]