
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

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1. (15 pts.) Let $f(x) = 3x(5 - x)$.

(a) Then the slope-predictor function for f at each x is given by

$m(x) =$

(b) It turns out that the graph of f has a horizontal tangent line for precisely one point on the graph of f, $(x_1,f(x_1))$. What is this ordered pair?

$(x_1,f(x_1)) =$

(c) For each real number x_0 that is different from x_1 in part (b) above, the tangent line to the graph of f at $(x_0,f(x_0))$ is not horizontal. As a consequence, it must pass through the x-axis at a unique point $(x_2,0)$. Obtain a formula for x_2 in terms of x_0.

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2. (5 pts.) Show how to use the squeeze law of limits to provide an evaluation of the following limit that is completely correct. You will need to show how to build a suitable inequality to provide a complete solution.

$$\lim_{x \to 0} x^2\sin(1/x) =$$

3. (5 pts.) It turns out that the slope-predictor function for the function $f(x) = \cos(2x)$ is the function $m(x) = -2 \cdot \sin(2x)$. Use this to obtain an equation for the line tangent to the graph of $f(x) = \cos(2x)$ at $x_0 = \pi/6$.

4. (10 pts.) (a) Using complete sentences and appropriate notation, state the theorem that is concerned with the intermediate value property of continuous functions.

(b) Apply the theorem concerning the intermediate value property of continuous functions to show that the given equation has a solution in the given interval.

$$x^3 + x + 1 = 0 \text{ on } [-1,0]$$

Explain completely. Deal with all the magical hypotheses.

5. (5 pts.) Find the value for the constant c, if possible, that will make the function f(x) defined below continuous at $x = \pi$. If you find such a c, using the definition, verify the continuity of f(x) at $x = \pi$. Suppose that

$$f(x) = \begin{cases} c^3 - x^3, & x \le \pi \\ c \cdot \cos(x/2), & x > \pi. \end{cases}$$

6. (20 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of page two if you run out of room here.]

(a) $\lim_{x \to +6} \dfrac{x + 6}{x^2 - 36} =$

(b) $\lim_{x \to -6} \dfrac{x + 6}{x^2 - 36} =$

(c) $\lim_{\theta \to 0} \dfrac{2}{\theta} \cdot \sin\left(\dfrac{3\theta}{\pi}\right) =$

(d) $\lim_{t \to 0} \dfrac{(t + 4)^{1/2} - 2}{t} =$

7. (10 pts.) Suppose that

$$h(x) = \begin{cases} x^2 - 3x & , \text{ if } x < -1 \\ 6 & , \text{ if } x = -1 \\ -4x & , \text{ if } x > -1 \end{cases}$$

Evaluate the following limits:

(a) $\lim_{x \to -1^+} h(x) =$

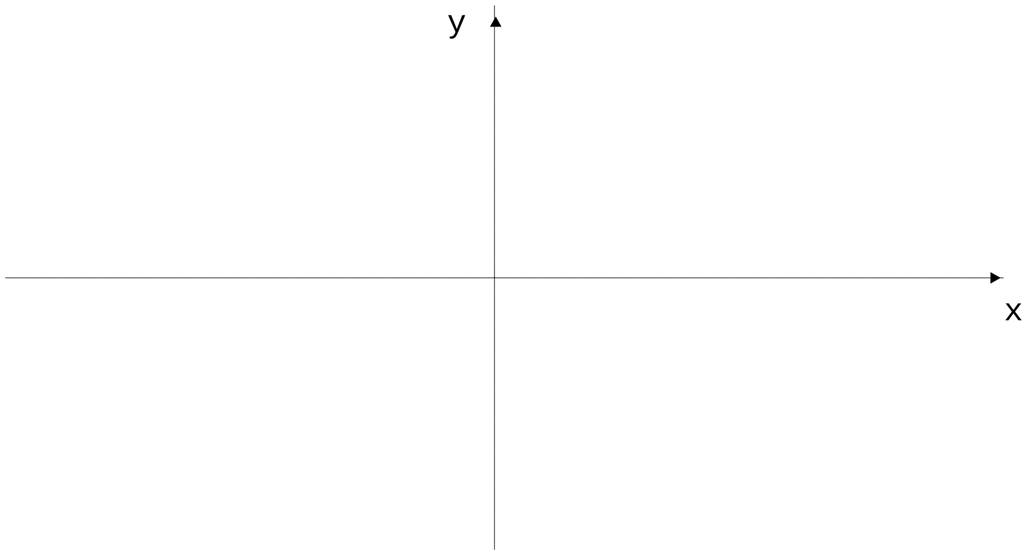
(b) $\lim_{x \to -1^-} h(x) =$

(c) What can you conclude from parts (a) and (b)? [Be as complete as possible. You should be able to discuss at least three concepts that have some bearing here.]

8. (10 pts.) Using an appropriate limit process, show completely how to obtain the slope-predictor function m(x) for the function $f(x) = 1/x$ for $x \neq 0$.

9. (10 pts.) First write the function $f(x) = |x - 1| - x$ in a piecewise-defined form below. Then sketch its graph below. Label carefully.

$f(x) =$



10. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definitions for each of the following items:

(a) $\lim_{x \to a} f(x) = L$ [Hint: This involves ε and δ.]//

(b) **Continuity** of a function $f(x)$ at a point $x = a$ //

11. (5 pts.) Give an ε - δ proof that $\lim_{x \to 2}(9x - 1) = 17$.