
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

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1. (10 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, $f'(x)$, of a function $f(x)$.

(b) Using the definition of the derivative as a limit, show all steps of the computation of $f'(x)$ when $f(x) = 1/(2x+1)$.

$f'(x) =$

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2. (10 pts.) (a) Using implicit differentiation, compute dy/dx and when $x^5 + y^2 = 9$. **Label your expressions correctly or else.**

(b) Obtain an equation for the line tangent to the graph of $x^5 + y^2 = 9$ at the point $(-1,-(10)^{1/2})$.

3. (10 pts.) (10 pts.) Locate and determine the maximum and minimum values of the function $f(x) = x^3 + 3x^2$ on the interval [-1, 1]. What magic theorem allows you to conclude that f(x) has a maximum and minimum even before you attempt to locate them? Why??

4. (5 pts.) Use logarithmic differentiation to find dy/dx when $y = x^{\tan(x)}$. **Label your expressions correctly or else.**

5. (5 pts.) Obtain the following limit. This is easy if you grok the definition of the derivative. [If not, break out your trigonometric tool kit!!]

$$\lim_{h \to 0} \frac{\cos((\pi/3) + h) - \cos(\pi/3)}{h} =$$

6. (20 pts.) Obtain the derivative of each of the following functions. You may use any of the rules of differentiation at your disposal. [4 pts./part]

(a) $f(x) = 4x^5 - 3x^{-6} + 7 \cdot \tan(x)$

$f'(x) =$

(b) $g(x) = (x^3 - 4x^{-1})\sec(x)$

$g'(x) =$

(c) $h(t) = \dfrac{3\cot(t)}{2t^5 - \csc(t)}$

$h'(t) =$

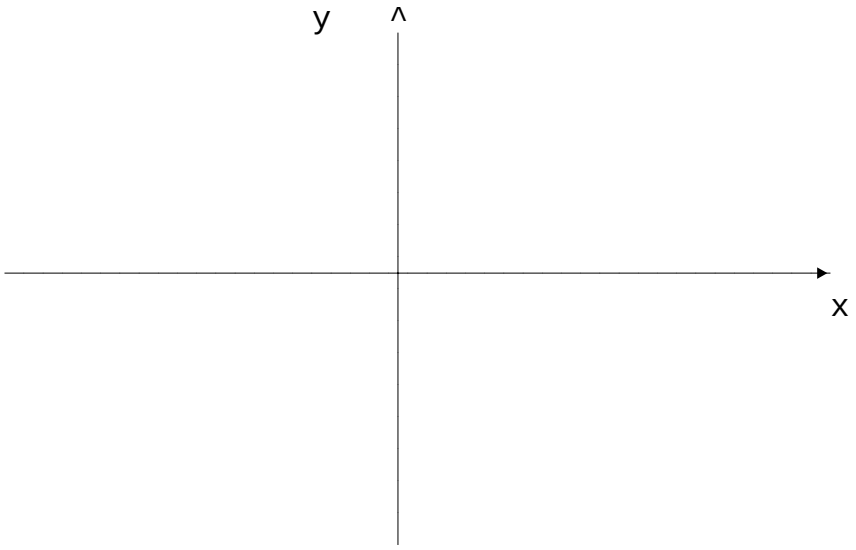
(d) $L(x) = \exp(3x^3 - 7x) - 2 \cdot \ln(5x^3 - 14)$

$L'(x) =$

(e) $y = 9^x + x^9 + 9^9 + \log_9(x) + \ln(9)$

$\dfrac{dy}{dx} =$

7. (15 pts.) (a) Sketch the graph of the function $f(x) = x + |x|$. (b) Then investigate its differentiability. Find the derivative where it exists. (c) Find the one-sided derivatives at the points where $f'(x)$ does not exist.



8. (5 pts.) Find all points in the interval $[-\pi,\pi]$ where the graph of $f(x) = x - 2\cdot\sin(x)$ has a horizontal tangent line.

9. (10 pts.) Suppose a person who is 5 feet tall is walking at night at a speed of 4 feet per second directly toward a street light that is 10 feet high. (a) How fast is the tip of the person's shadow moving along the ground? (b) At what rate is the person's shadow decreasing in length??

10. (10 pts.) A commuter train carries 600 passengers each day from a suburb to a city. It costs $1.50 per person to ride the train. Market research reveals that 40 fewer people would ride the train for each 5¢ increase in fare, 40 more for each 5¢ decrease in fare. Show completely how to determine which fare should be charged to yield the largest possible revenue, and give the fare that does yield the largest possible revenue. [Warning: Details, details, details...]