
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

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1. (10 pts.) (a) Using implicit differentiation, compute dy/dx and d^2y/dx^2 when $\sin(y) = xy$. **Label your expressions correctly or else.**

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2. (5 pts.) Use a linear approximation $L(x)$ to an appropriate function $f(x)$, with an appropriate value of a, to estimate the value of $65^{-2/3}$.

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3. (5 pts.) Write dy in terms of x and dx when $y = \sin(2x)\cos(3x)$.

$dy \quad =$

4. (4 pts.) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

5. (16 pts.) Fill in the blanks of the following analysis with the correct terminology.

Let $f(x) = 4x^3 - x^4$. Then $f'(x) = -4x^3 + 12x^2 = -4x^2(x - 3)$.

Since $f'(x) > 0$ when $0 < x < 3$ or $x < 0$, and f is continuous, f is ____________________ on the interval $(-\infty, 3)$. Also, because $f'(x) < 0$ for $3 < x$, f is ____________________ on the set $(3,\infty)$. Obviously, $x = 0$ and $x = 3$ are ____________________ points of f since $f'(0) = 0$ and $f'(3) = 0$. By using the first derivative test, it follows easily that f has ______________________________________ at $x = 0$, and _______________________________________ at $x = 3$.

Since $f''(x) = -12x^2 + 24x = -12x(x - 2)$, we have $f''(0) = 0$, $f''(2) = 0$, $f''(x) > 0$ when $0 < x < 2$, and $f''(x) < 0$ when $x > 2$ or $x < 0$. Thus, f is _________________________________ on the set $(-\infty, 0) \cup (2,\infty)$, f is ________________________________ on the interval $(0,2)$, and f has _____________________________ at $x = 0$ and $x = 2$.

6. (10 pts.) (a) Find all the critical points of the function $f(x) = 3 \cdot (x^2 - 2x)^{1/3}$. (b) Apply the second derivative test at each critical point, c, where $f'(c) = 0$, and draw an appropriate conclusion.

7. (5 pts.) Suppose $f(x) = x^{1/2}$ on the interval [100,101]. Show that there is a number c in (100,101) such that

$$101^{1/2} = 10 + (1/2)c^{-1/2}$$

by using the Mean Value Theorem. [Take care to tell me about the hypotheses that must be satisfied before you assert the conclusion is true for the function at hand.]

8. (5 pts.) Find the function f(x) that satisfies the following two equations: $f'(x) = 2 \cdot \sec^2(x) + (4/\pi)$ for every real number x in $(-\pi/2, \pi/2)$, and $f(\pi/4) = 8$.

9. (10 pts.) Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible.

(a) $\lim_{x \to \infty} ((x^2 + 18x)^{1/2} - x) =$

(b) $\lim_{x \to 0} \dfrac{\exp(x^3) - 1}{x - \sin(x)} =$

10. (10 pts.) Evaluate each of the following anti-derivatives.

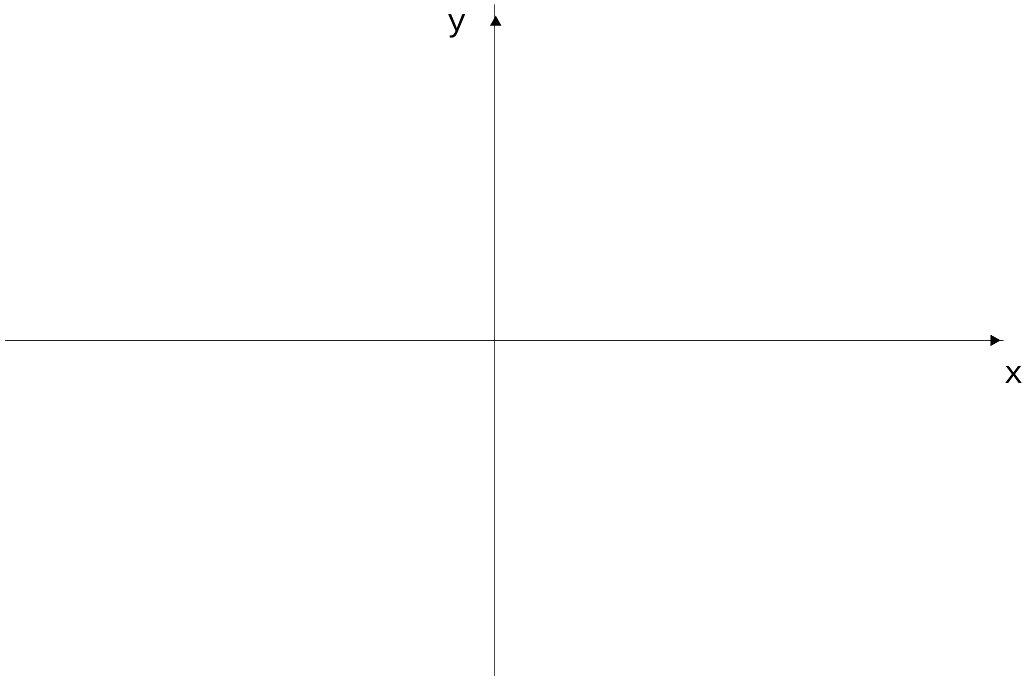
(a)

$$\int 7x^6 - \frac{\pi}{x} - \frac{12}{x^3} + 5 \cdot \cos(10x)\, dx =$$

(b)

$$\int e^{2x} + e^{-2x} - \frac{2x^4 - 3x^3 + 5}{7x^2}\, dx =$$

11. (10 pts.) Sketch the graph of $f(x) = 2x^2 - x^4$. At the very least you should determine the critical points of f and intervals where f is increasing or decreasing to do this. [If you run out of room here in doing your analysis, work on the back of Page 4.]



12. (10 pts.) Determine the open intervals where the funnction $f(x) = x^3(x - 1)^4$ is concave up and concave down. Then locate any inflection points. [Note: Do not attempt to graph this varmint. Communicate your results using complete sentences.]

Silly Ten Point Bonus: Pretend that f is differentiable in an open interval I that contains the number c with $f'(c) = 0$. Suppose that $f''(c) < 0$. By unraveling the definition of $f''(c)$ in terms of epsilons and deltas, show that there is a $\delta > 0$ such that $f'(c + h)$ and h have different signs if $0 < |h| < \delta$. What does this imply about the signs of $f'(x)$ near c. [Details!! Work on the back of Page 3. You do not have room here.]