Student Number: **Exam Number:**

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Since the answer really consists of all the magic transformations and incantations, do not "box" your final results.*

1. (30 pts.) Very basic derivatives .... Provide the first derivative for each of the following functions.

(a) $f(x) = \cos^{-1}(x)$ $f'(x) =$

(b) $f(x) = x^{53}$ $f'(x) =$

(c) $f(x) = \sec(x)$ $f'(x) =$

(d) $f(x) = \ln(53)$ $f'(x) =$

(e) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(f) $f(x) = \cos(x)$ $f'(x) =$

(g) $f(x) = \tan^{-1}(x)$ $f'(x) =$

(h) $f(x) = \cot(x)$ $f'(x) =$

(i) $f(x) = \csc(x)$ $f'(x) =$

(j) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(k) $f(x) = \sin(x)$ $f'(x) =$

(l) $f(x) = 32^{x}$ $f'(x) =$

(m) $f(x) = \tan(x)$ $f'(x) =$

(n) $f(x) = e^{x}$ $f'(x) =$

(o) $f(x) = \log_5(x)$ $f'(x) =$

2. (30 pts.) Very basic antiderivatives .... Give each of the following antiderivatives. **Do not forget the arbitrary constant.**

(a) $\int \sin(x)\,dx =$

(b) $\int x^{50}\,dx =$

(c) $\int e^{x}\,dx =$

(d) $\int \csc(x)\cdot\cot(x)\,dx =$

(e) $\int \cos(x)\,dx =$

(f) $\int \frac{1}{1+x^2}\,dx =$

(g) $\int \sec(x)\cdot\tan(x)\,dx =$

(h) $\int x^{-1}\,dx =$

(i) $\int \frac{1}{(1-x^2)^{1/2}}\,dx =$

(j) $\int \frac{1}{|x|(x^2-1)^{1/2}}\,dx =$

(k) $\int \csc^2(x)\,dx =$

(l) $\int \sec^2(x)\,dx =$

(m) $\int \ln(53)\,dx =$

(n) $\int 5^{x}\,dx =$

(o) $\int 0\,dx =$

3. (20 pts.) Some basic limits .... For each of the following, find the limit if it exists. If the limit does not exist, say so. Be as precise as possible in doing this. [Warning: At least one of these involves the definition of the derivative!!]

(a) $\lim_{x \to 5^-} \dfrac{x^2 - 49}{7x - 49} =$

(b) $\lim_{x \to -\infty} \dfrac{4 + 3x^4 - 9x^2}{8x^5 - 11x + 11} =$

(c) $\lim_{\theta \to 0} \dfrac{\tan(6\pi^3\theta)}{\sin(3\pi^4\theta)} =$

(d) $\lim_{\theta \to 0} \dfrac{\tan((\pi/4) + \theta) - 1}{\theta} =$

(e) $\lim_{x \to -\infty} 2 \cdot \cot^{-1}(x) =$

4. (20 pts.) More devious derivatives to do ... .
Compute the first derivative of each of the following functions.
Do not simplify the algebra.

(a) $f(x) = 10 \cdot x^{1/5} - \tan^{-1}(x) + 30 \cdot \cos(\pi/4) + \sec(x^4) - 8^{\ln(\pi/4)}$

$f'(x) =$

(b) $y = (3x^4 - 2) \cdot e^{\tan(x^3)}$

$\frac{dy}{dx} =$

(c) $g(\theta) = \frac{\cos(\theta) - 4 \cdot e^{\cot(\theta)}}{\theta^4}$

$\frac{dg(\theta)}{d\theta} =$

(d) $h(t) = \log_{52}(4t) - 3^{52t} + (52 \cdot t^3)^2 + \ln(52) \cdot x$

$h'(t) =$

5. (25 pts.) More ignominious integrals .... Evaluate each of the following antiderivatives or indefinite integrals.

(a) $\int 8x^7 - \frac{7}{x} - 5\cdot\cos(5x)\, dx =$

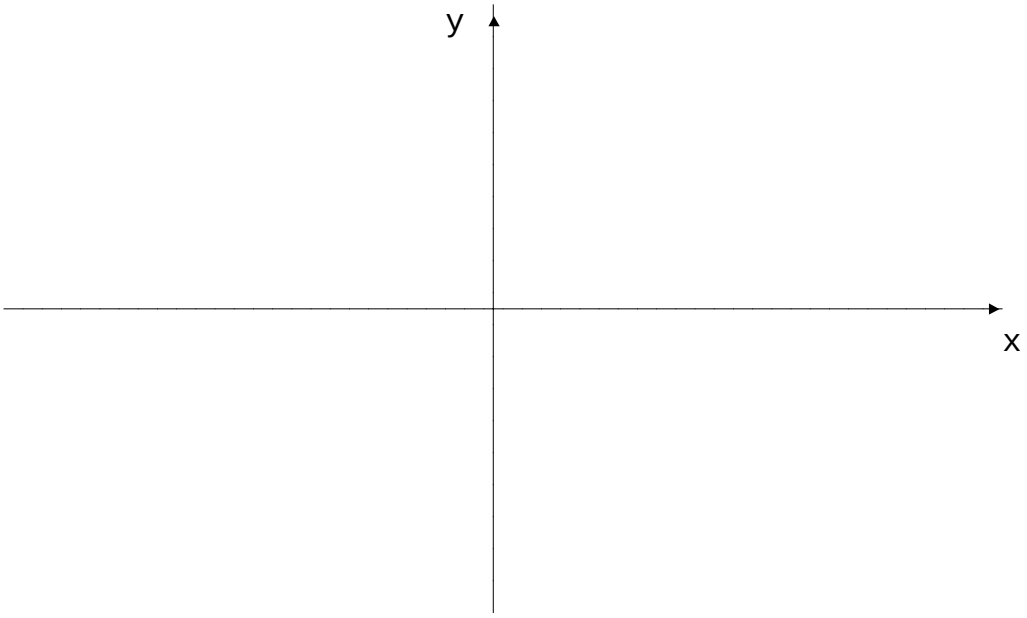
(b) $\int e^{3x+3} - \frac{52}{x^2+4} + \frac{20x}{x^2+4}\, dx =$

(c) $\int 6\cdot\tan^5(\theta)\cdot\sec^2(\theta)\, d\theta =$

(d) $\int (8t+4)\cdot(t^2+t+1)\, dt =$

(e) $\int \cos^3(x)\, dx =$

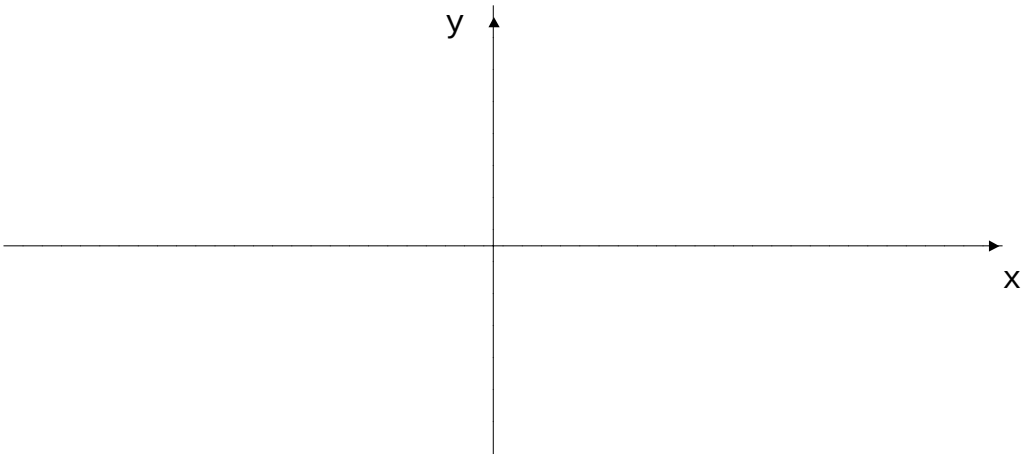
6. (10 pts.) Let $f(x) = 4x^3 - 3x^4$. Provide a complete analysis of this function on the back of Page 5. Then, below, plot the zero(s), the critical point(s), and the inflection point(s) of f. Finally, connect the dots and display the behavior of f at $\pm\infty$. Label very carefully.



7. (10 pts.) A 10 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 1 feet per second, how fast will the top of the ladder be moving down the wall when it is 3 feet above the ground??

8. (5 pts.) Give an equation for the line tangent to the graph of $f(x) = \tan(6x)$ at $x = -\pi/24$.

9. (5 pts.) Sketch the graph of $y = \cot^{-1}(x)$ below. Label very carefully.



10. (10 pts.) Obtain the maximum and minimum values attained by $f(x) = x^2 + (16/x)$ on the closed interval $[1,3]$.

11. (10 pts.) Provide a complete evaluation each of the following limits. In particular, point out any use of L'Hopital's Rule, and when *squeezing*, give a complete argument.

(a) $\lim_{x \to \infty} \dfrac{32x^3 + \sin(\pi x)}{\cos(\pi x) - x^3} =$

(b) $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{4 - 4\cos(2x)} =$

12. (5 pts.) Find a formula for the function g(x) which satisfies the following: $g'(x) = e^x/(1 + e^{2x})$ and $g(0) = 8\pi$.

13. (5 pts.) Use logarithmic differentiation to find dy/dx when

$$y = 5^{\sin(x)}\sin^{\ln(\tan(x))}.$$

Note: This is ill-formed. What was intended is obvious. Fix the function. Then differentiate.

14. (10 pts.) A cattle ranch currently allows 20 steers per acre of grazing land. As a consequence, on average its steers weigh 2000 pounds at market. Crude estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 pound for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market weight for its cattle.

15. (5 pts.) Give a complete ε - δ proof that

$$\lim_{x \to -2} (20x - 5) = -45.$$

Place your work on the back of Page 7 of 8.

20 Point Bonus: Pretend that f is differentiable in an open interval I that contains the number c with $f'(c) = 0$. Suppose that $f''(c) > 0$. By unraveling the definition of $f''(c)$ in terms of epsilons and deltas, show that there is a $\delta > 0$ such that $f'(c + h)$ and h have the same signs if $0 < |h| < \delta$. What does this imply about the signs of $f'(x)$ for values of x near c??? And what does this imply about local extrema. Details, please. All the magic should be revealed. Say where, here: