
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

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1. (10 pts.) Using the text's four step process, show completely how to obtain the slope-predictor function m(x) for the function $f(x) = 1/x^2$ for $x \neq 0$.

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2. (10 pts.) Let $f(x) = 2x(x + 3)$.

(a) Then the slope-predictor function for f is given by

$m(x) =$

(b) It turns out that the graph of f has a horizontal tangent line at precisely one point on the graph of f, $(x_1, f(x_1))$. What is this ordered pair?

$(x_1, f(x_1)) =$

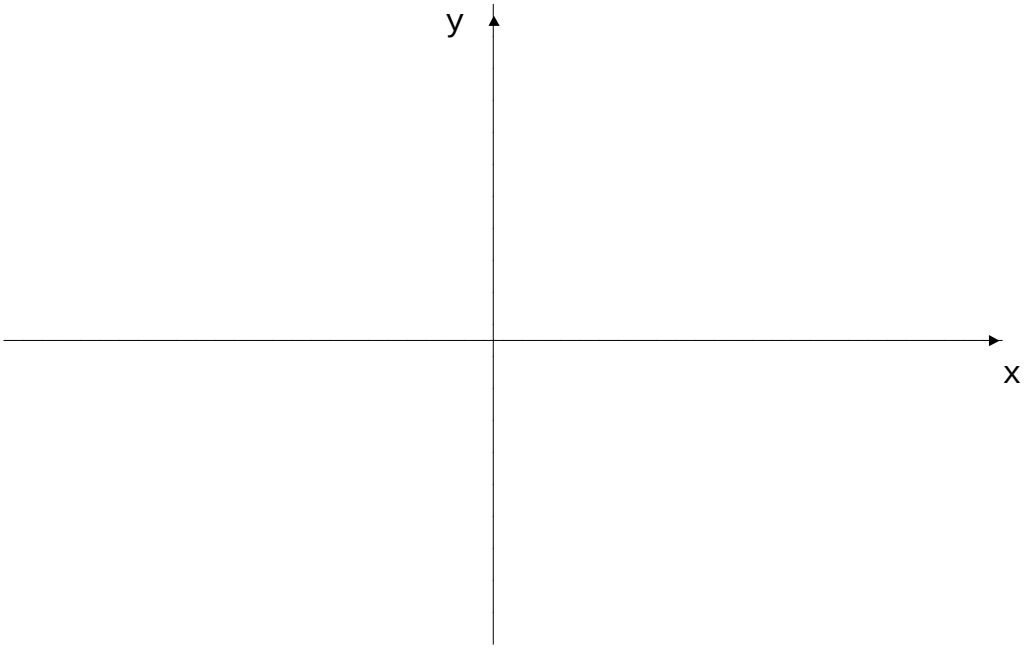
3. (5 pts.) It turns out that the slope-predictor function for the function $f(x) = \tan(x)$ is the function $m(x) = \sec^2(x)$. Use this to obtain an equation for the line tangent to the graph of $f(x) = \tan(x)$ at $x_0 = \pi/4$.

4. (15 pts.) (a) First write the function

$$f(x) = \frac{x^2 - 1}{|x - 1|}$$

in a piecewise-defined form below. (b) Then sketch its graph. Label carefully. (c) Evaluate each one-sided limit at $x = 1$. (d) Using part (c), what can you say about the existence of the two-sided limit at $x = 1$??

$f(x) =$



$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1^-} f(x) =$

5. (20 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of page two if you run out of room here.]

(a) $\lim_{x \to -9} \dfrac{x + 9}{x^2 - 81} =$

(b) $\lim_{x \to +9} \dfrac{x + 9}{x^2 - 81} =$

(c) $\lim_{\theta \to 0} \dfrac{\tan(5\pi\theta)}{\sin(2\theta)} =$

(d) $\lim_{x \to 4} \dfrac{x - 4}{2 - x^{1/2}} =$

6. (10 pts.) Where does the straight line tangent to the graph of $y = x^2$ at the point (-10,100) intersect the x-axis??

7. (5 pts.) Using complete sentences and appropriate notation, provide the precise mathematical definitions for each of the following items:

$\lim_{x \to a} f(x) = L$ [Hint: This involves ε and δ.]//

8. (5 pts.) Give a complete ε - δ proof that

$$\lim_{x \to -3}(7x - 9) = -30.$$

9. (5 pts.) Show how to use the squeeze law of limits to provide an evaluation of the following limit that is completely correct. You will need to show how to build a suitable inequality to provide a complete solution.

$\lim_{x \to 0} x^2 \cos^2(1/x) =$

10. (5 pts.) Suppose that

$$h(x) = \begin{cases} x^2 - 3x & , \text{ if } x < 1 \\ 6 & , \text{ if } x = 1 \\ -2x & , \text{ if } x > 1 \end{cases}$$

Evaluate the following limits:

(a) $\lim_{x \to 1^+} h(x) =$

(b) $\lim_{x \to 1^-} h(x) =$

(c) What can you conclude from parts (a) and (b)? Why??

11. (5 pts.) Show a complete evaluation of the following somewhat thorny limit:

$$\lim_{\theta \to 0} \frac{1 - \cos(\theta)}{\theta \cdot \sin(2\theta)} =$$

12. (5 pts.) What is the slope of the line normal to the curve $y = 2x^2 + 3x - 5$ at the point P(2,9) ??