
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

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1. (10 pts.) (a) Using implicit differentiation, compute dy/dx and when $x^3 + y^2 = 9$. **Label your expressions correctly or else.**

(b) Obtain an equation for the line tangent to the graph of $x^3 + y^2 = 9$ at the point $(-1,-(10)^{1/2})$.

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2. (5 pts.) Use a linear approximation L(x) to an appropriate function f(x), with an appropriate value of a, to estimate the value of ln(11/10).

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3. (5 pts.) Write dy in terms of x and dx when $y = \sin(2x)\cos(3x)$.

dy =

4. (5 pts.) If you want to find the point (x_0,y_0) on the line defined by $2x + y = 3$ that is closest to the point (3,2), what function should you minimize on what domain?? [Note: Simply obtain the function and its domain. DON'T attempt to analyze the function.]

5. (5 pts.) Use logarithmic differentiation to find dy/dx when $y = x^{\ln(x)}$. **Label your expressions correctly or else.**

6. (5 pts.) Find the function f(x) that satisfies the following two equations: $f'(x) = x^{-2}$ and $f(1) = 1$.

7. (5 pts.) Compute the differential of each side of the given equation, regarding x and y as dependent variables (as if both were functions of some third, unspecified, variable). Then solve for dy/dx.

$$x^3 + y^3 = 3xy$$

8. (10 pts.) Differentiate each of the following functions. Do not attempt to simplify the algebra. **Label your derivatives correctly or else.**

(a) $g(t) = [\ln(t) + e^{-t}]/\sin(3t)$

(b) $f(t) = t \cdot \sec(t) \cdot \tan(t)$

9. (5 pts.) Find all points in the interval $[0,2\pi]$ where the graph of $f(x) = x - 2 \cdot \cos(x)$ has a horizontal tangent line.

10. (5 pts.)
Suppose $f(x) = x^{1/2}$ on the interval [81,82]. Show that there is a number c in (81,82) such that

$$82^{1/2} = 9 + (1/2)c^{-1/2}$$

by using the Mean Value Theorem. [Take care to tell me about the hypotheses that must be satisfied before you assert the conclusion is true for the function at hand. How the incantation is muttered is significant.]

11. (10 pts.) The height of a cone is decreasing at 3 cm/sec. while its radius is increasing at 2 cm/sec. When the radius is 8 cm and the height is 12 cm, what is the rate at which the volume is changing? Is the volume increasing or decreasing then?? Which??? [Hint: $V = (1/3)\pi r^2 h$]

12. (10 pts.) Find and classify the critical points of the function $f(x) = \sin^3(x)$ in the interval $(-3,3)$.

13. (10 pts.) Show that the equation $x \cdot \ln(x) = 3$ has exactly one solution in the interval $[2,4]$.

14. (10 pts.) Use the first derivative to determine the open intervals where the polynomial function $f(x) = (x - 1)^3(x - 2)^4$ is increasing and decreasing. Be specific.

Silly Ten Point Bonus: Let $f(\theta) = \sin^\circ(\theta)$ and $g(\theta) = \cos^\circ(\theta)$ be your friendly old sine and cosine functions whose argument θ is measured in *degrees*. Compute $f'(\theta)$ and $g'(\theta)$.