Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

1. (10 pts.) Pretend y is a function of x. Using implicit differentiation, compute dy/dx and d^2y/dx^2 when $y^2 - x^2 = 25$. **Label your expressions correctly or else.**

2. (10 pts.) Compute the first derivative of each of the following functions.

(a) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(b) $f(x) = \cos^{-1}(x)$ $f'(x) =$

(c) $f(x) = \cot^{-1}(x)$ $f'(x) =$

(d) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(e) $f(x) = \tan^{-1}(x)$ $f'(x) =$

3. (16 pts.) Fill in the blanks of the following analysis with the correct terminology.

Let $f(x) = 8x^3 - x^4$. Then $f'(x) = -4x^3 + 24x^2 = -4x^2(x - 6)$.

Consequently, $x = 0$ and $x = 6$ are ____________________ points of f. Since $f'(x) < 0$ for $6 < x$, f is ____________________ on the set $(6,\infty)$. Also, because $f'(x) > 0$ when $0 < x < 6$ or $x < 0$, and f is continuous, f is ____________________ on the interval $(-\infty , 6)$. Using the first derivative test, it follows that f has a(n) ______________________________________ at $x = 6$, and _______________________________________ at $x = 0$.

Since $f''(x) = -12x^2 + 48x = -12x\cdot(x - 4)$, we have $f''(0) = 0$, $f''(4) = 0$, $f''(x) > 0$ when $0 < x < 4$, and $f''(x) < 0$ when $x > 4$ or $x < 0$. Thus, f is ___________________________ on the interval $(0,4)$, f is _________________________ on the set $(-\infty ,0) \cup (4,\infty)$, and f has _____________________________ at $x = 0$ and $x = 4$.

4. (4 pts.) Find the function f(x) that satisfies the following two equations: $f'(x) = 4\sec^2(x) + (4/\pi)$ for every real number x in $(-\pi/2,\pi/2)$, and $f(\pi/4) = 16$.

5. (10 pts.) (a) Find all the critical points of the function $f(x) = 3(x^2 - 10x)^{1/3}$. (b) Apply the second derivative test at each critical point, c, where $f'(c) = 0$, and draw an appropriate conclusion.

6. (10 pts.) Evaluate each of the following anti-derivatives.

(a)

$$\int 8x^7 - \frac{1}{x} - \frac{12}{x^3} + 20 \cdot \cos(10x)\ dx =$$

(b)

$$\int e^x + e^{-x} - \frac{x^4 - 14x^3 + 14}{x^2} + \frac{1}{1 + x^2}\ dx =$$

9. (20 pts.) Limits to lament. Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible. For some of these, L'Hopital's Rule may prove useful.

(a)

$$\lim_{x \to \infty} \frac{2x^2 - 1}{5x^2 + 3x} =$$

(b)

$$\lim_{x \to 0} \frac{\sin(5x)}{\tan(3x)} =$$

(c)

$$\lim_{x \to \pi/2} \frac{4x - 2\pi}{\tan(2x)} =$$

(d)

$$\lim_{x \to 0} \frac{1}{x} \ln\left(\frac{4x + 8}{7x + 8}\right) =$$

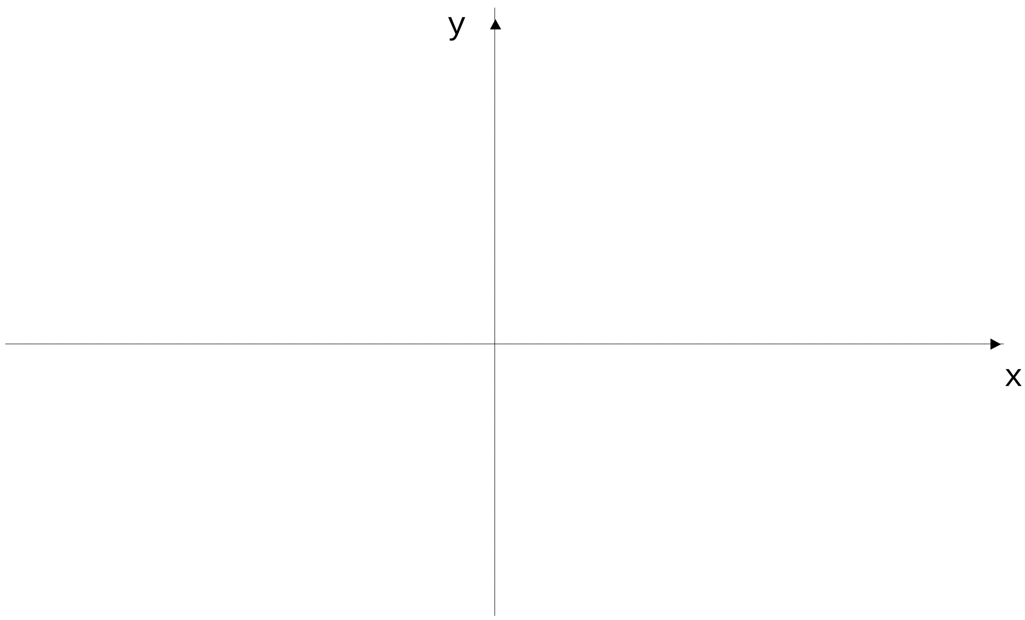
(e) Suppose that A > 0 is a real number. Then

$$\lim_{x \to \infty} ((x^2 + Ax)^{1/2} - x) =$$

11. (10 pts.) Very carefully sketch the following function. Use all the data provided and label very carefully.

f(x) is continuous on $\mathbb{R}$ and satisfies the following:

(1) $f(0) = 1$;
(2) $x \neq 0 \Rightarrow f'(x) < 0$;
(3) $\lim_{x \to 0^-} f'(x) = -\infty$ and $\lim_{x \to 0^+} f'(x) = -\infty$;
(4) $x < 0 \Rightarrow f''(x) < 0$ and $x > 0 \Rightarrow f''(x) > 0$
(5) $f(x) \longrightarrow 2$ as $x \longrightarrow -\infty$, and $f(x) \longrightarrow 0$ as $x \longrightarrow \infty$.



12. (10 pts.)
Determine where the function $f(x) = x^4 - 6x^2$ is concave up, concave down, and locate any inflection points it may have.

Silly Ten Point Bonus: Suppose the generic cubic function $f(x) = ax^3 + bx^2 + cx + d$ with $a \neq 0$ has exactly two distinct real critical points. Must this varmint have an inflection point exactly midway between these critical points? Proof?? [Details!! Work on the back of Page 4. You do not have room here.]