**Student Number:** **Exam Number:**

**Read Me First:** *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Since the answer really consists of all the magic transformations and incantations, do not "box" your final results.*

1. (30 pts.) Very basic derivatives .... Provide the first derivative for each of the following functions.

(a) $f(x) = \cos(x)$ $f'(x) =$

(b) $f(x) = \ln(54\pi)$ $f'(x) =$

(c) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(d) $f(x) = x^{54}$ $f'(x) =$

(e) $f(x) = \sin(x)$ $f'(x) =$

(f) $f(x) = \cos^{-1}(x)$ $f'(x) =$

(g) $f(x) = \tan(x)$ $f'(x) =$

(h) $f(x) = \cot(x)$ $f'(x) =$

(i) $f(x) = \ln(x)$ $f'(x) =$

(j) $f(x) = \sec(x)$ $f'(x) =$

(k) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(l) $f(x) = e^x$ $f'(x) =$

(m) $f(x) = \tan^{-1}(x)$ $f'(x) =$

(n) $f(x) = 4^x$ $f'(x) =$

(o) $f(x) = \log_5(x)$ $f'(x) =$

2. (30 pts.) Very basic antiderivatives .... Give each of the following antiderivatives. **Do not forget the arbitrary constant.**

(a) $\int \cos(x)\,dx =$

(b) $\int 5^x\,dx =$

(c) $\int \csc(x)\cdot\cot(x)\,dx =$

(d) $\int 0\,dx =$

(e) $\int \sin(x)\,dx =$

(f) $\int \frac{1}{|x|(x^2 - 1)^{1/2}}\,dx =$

(g) $\int \tan(\pi/4)\,dx =$

(h) $\int x^{-1}\,dx =$

(i) $\int \frac{1}{1 + x^2}\,dx =$

(j) $\int \frac{1}{(1 - x^2)^{1/2}}\,dx =$

(k) $\int \sec(x)\cdot\tan(x)\,dx =$

(l) $\int \sec^2(x)\,dx =$

(m) $\int e^x\,dx =$

(n) $\int x^{54}\,dx =$

(o) $\int \csc^2(x)\,dx =$

3. (20 pts.) Some basic limits .... For each of the following, find the limit if it exists. If the limit does not exist, say so. Be as precise as possible in doing this. [Warning: At least one of these involves the definition of the derivative!!]

(a) $\lim_{x \to 4} \frac{x-4}{x^2-3x-4} =$

(b) $\lim_{x \to -1} \frac{x-4}{x^2-3x-4} =$

(c) $\lim_{t \to \infty} \frac{4-2t^4+3t^2}{3t^4-11t+11} =$

(d) $\lim_{\theta \to 0} \frac{\sin(3\theta)}{\tan(7\theta)} =$

(e) $\lim_{\theta \to 0} \frac{\ln(5+\theta)-\ln(5)}{\theta} =$

4. (20 pts.) More devious derivatives to do ... .
Compute the first derivative of each of the following functions.
Do not simplify the algebra.

(a) $f(x) = 6 \cdot x^4 - 20 \cdot \cos(x) + 30 \cdot \sec^{-1}(x^2)$

$f'(x) =$

(b) $y = \sin(3 \cdot x - 4) \cdot e^{x^2}$

$\frac{dy}{dx} =$

(c) $g(\theta) = \frac{\tan(\theta) - 4 \cdot \sec(\theta)}{\ln(\theta)}$

$\frac{dg(\theta)}{d\theta} =$

(d) $h(t) = \log_2(t) - 3 \cdot 2^t + (4 \cdot t^2)^3 + \ln(2)$

$h'(t) =$

(e) $L(x) = \sin^{-1}(e^x)/\tan^{-1}(\ln(x))$

$L'(x) =$

5. (25 pts.) More ignominious integrals .... Evaluate each of the following antiderivatives or indefinite integrals.

(a) $\int 5x^4 - \frac{8}{x} - \sin(x)\, dx =$

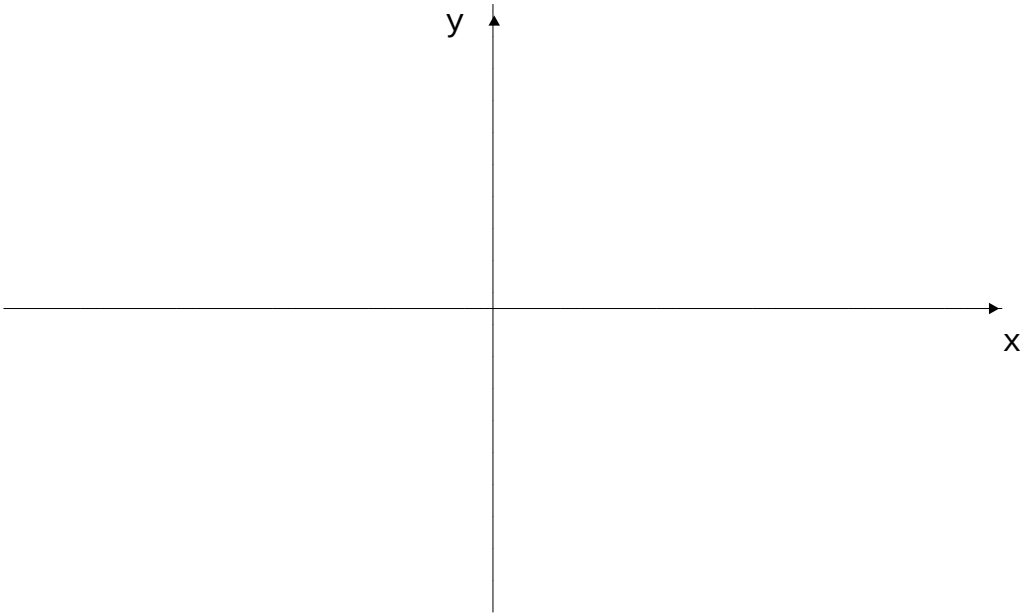
(b) $\int e^{7x - 5} - \frac{10 \cdot x}{5x^2 + 1}\, dx =$

(c) $\int 2 \cdot \sin(\theta) \cdot \cos(\theta) - \frac{25}{(1 - 4\theta^2)^{1/2}}\, d\theta =$

(d) $\int \frac{1}{1 + 9t^2} + (6t + 21)(t^2 + 7t)^{7/8}\, dt =$

(e) $\int \sin^3(x)\, dx =$

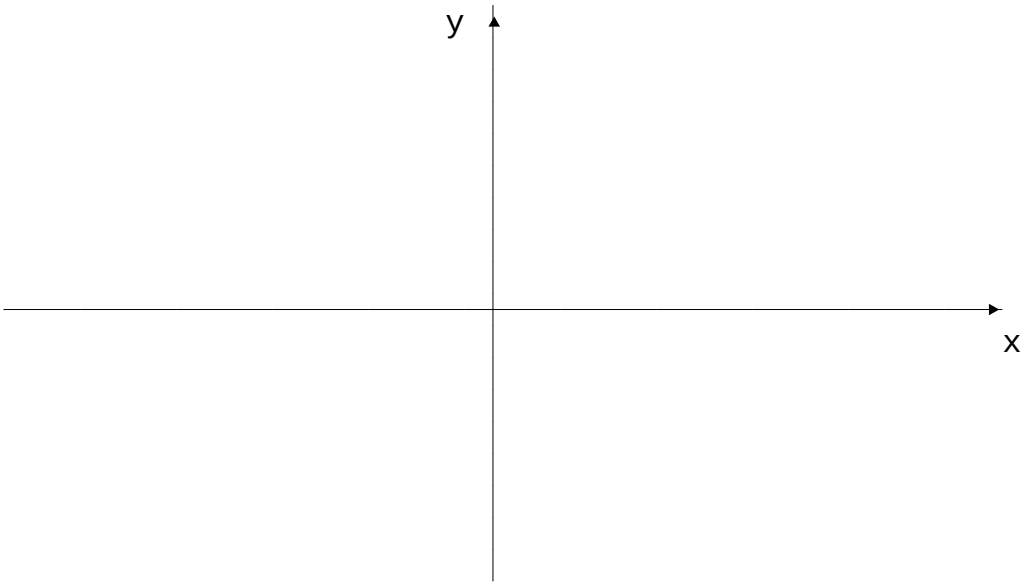
6. (10 pts.) Let $f(x) = 2x/(x^2 + 1)$. Provide a complete analysis of this function on the back of Page 5. Then, below, plot the zero(s), the critical point(s), and the inflection point(s) of f. Finally, connect the dots and display the limit behavior of f at $\pm\infty$. Label very carefully.




7. (10 pts.) A box with a square base, vertical sides, and an open top is made from 300 square feet of material. Find the dimensions yielding the greatest volume.

8. (5 pts.) Give an equation for the line tangent to the graph of the curve defined by the parametric equations $x = 2t^2 + 1$ and $y = 3t^2 + 2$ when $t = 1$.

9. (5 pts.) Suppose a curve is given by the parametric equations $x = 5\cdot\cos(t)$ and $y = 3\cdot\sin(t)$. Eliminate the parameter and then sketch the curve below.




10. (10 pts.) A person 6 ft. tall is walking at a rate of 3 ft./sec. toward a streetlight 24 feet high.
(a) At what rate is his shadow length changing?
(b) How fast is the tip of his shadow moving??
[Hints: (1) Draw a picture. (2) Look for similar triangles.]

11. (10 pts.) Provide a complete evaluation each of the following limits. In particular, point out any use of L'Hopital's Rule.

(a) $\lim_{x\to\infty} x^2 e^{-x} =$

(b) $\lim_{x\to\pi/2} \tan(x)\cos(3x) =$

12. (5 pts.) Find a formula for the function g(x) which satisfies the following: $g'(x) = \sec(x)\tan(x) + x$ ; $g(0) = 4\pi$.

13. (5 pts.) Use logarithmic differentiation to find dy/dx when

$$y = (10^{\sin(x)})/(3^{\ln(27x + 2)}).$$

14. (10 pts.) Find the maximum and minimum values of the function $f(x) = x^{2/3}(20 - x)$ on the interval [-1,20].

15. (5 pts.) Give a complete $\varepsilon$ - $\delta$ proof that

$$\lim_{x \to 2} (2x - 5) = -1.$$

**Place your work on the back of Page 7 of 8.**

**20 Point Bonus:** Show how to use the Mean Value Theorem to estimate the value of $(101)^{1/2}$ and the amount of error in your approximation.