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**Read Me First:** *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Since the answer really consists of all the magic transformations, do not "box" your final results. Communicate. Show me all the magic on the page.*

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1. (10 pts.) Using the text's four step process, show completely how to obtain the slope-predictor function m(x) for the function $f(x) = x^3$.

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2. (15 pts.) Let $f(x) = 2x(x - 4)$.

(a) Then the slope-predictor function for f is given by

$m(x) =$

(b) It turns out that the graph of f has a horizontal tangent line at precisely one point on the graph of f, $(x_1, f(x_1))$. What is this ordered pair?

$(x_1, f(x_1)) =$

(c) What is the slope of the line perpendicular to the line that is tangent to the graph of f at the point P(4,0) ?? Thus, find the slope of the line normal to the graph at P(4,0).

3. (25 pts.) For each of the following, find the limit if the limit exists. If the limit fails to exist, say so. Be as precise as possible here. [Work on the back of Page 1 of 4 if you run out of room here.]

(a) $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 4} =$

(b) $\lim_{x \to 9} \frac{x - 9}{3 - x^{1/2}} =$

(c) $\lim_{x \to 0} \frac{x \cdot \sin(2x)}{1 - \cos(x)} =$

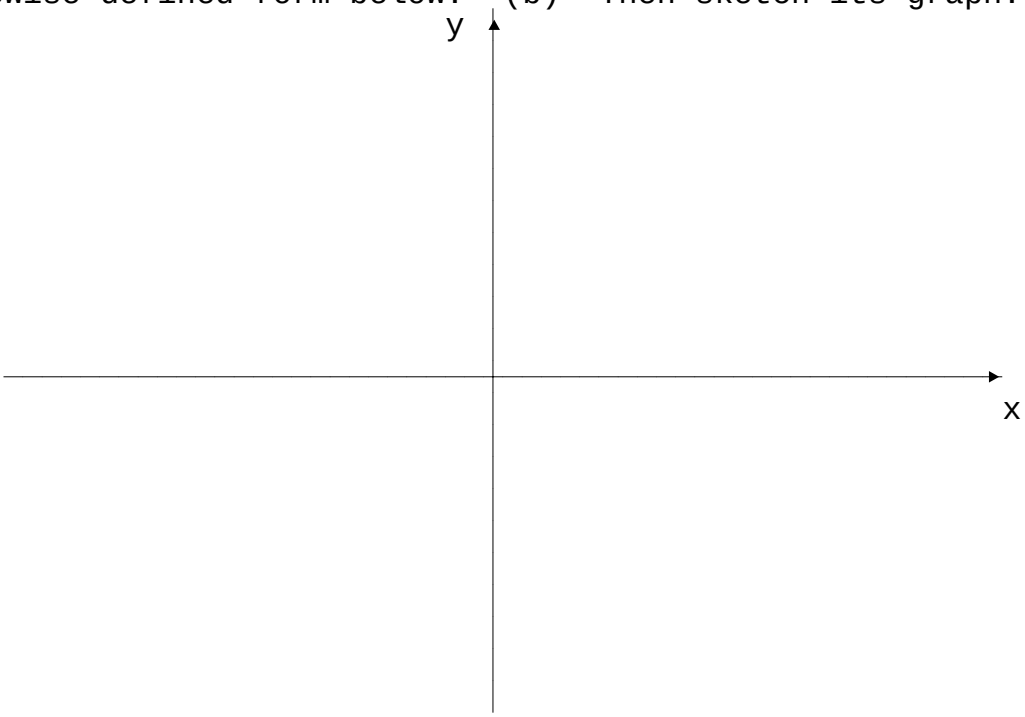
(d) $\lim_{x \to 5^-} \frac{2x - 10}{|x - 5|} =$

(e) $\lim_{x \to 1} (4x + 5)^{1/2} =$

4. (10 pts.) (a) First write the function

$$f(x) = \frac{x^2 - 4}{|x - 2|}$$

in a piecewise-defined form below. (b) Then sketch its graph.




5. (15 pts.) Suppose that

$$h(x) = \begin{cases} x^2 - 2x, & \text{if } x > 1 \\ 2, & \text{if } x = 1 \\ 2x - 2, & \text{if } x < 1 \end{cases}$$

Evaluate each of the following limits.

(a) $\lim_{x \to 1^+} h(x) =$

(b) $\lim_{x \to 1^-} h(x) =$

(c) $\lim_{x \to 2} h(x) =$

(d) $\lim_{x \to -2} h(x) =$

(e) $\lim_{x \to 1} h(x) =$

6. (5 pts.) Using complete sentences and appropriate notation, provide the precise ε - δ mathematical definition of

$$\lim_{x \to a} f(x) = L$$

7. (5 pts.) Show how to use the squeeze law of limits to provide an evaluation of the following limit that is completely correct. You will need to show how to build a suitable inequality to provide a complete solution.

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) =$$

8. (5 pts.) It turns out that the slope-predictor function for the function f(x) = sec(x) is the function m(x) = sec(x)tan(x). Use this to obtain an equation for the line tangent to the graph of f(x) = sec(x) at $x_0 = \pi/4$.

9. (5 pts.) The line tangent to the graph of $y = x^2$ at $x = x_0$ is given by the equation

$$y - (x_0)^2 = 2x_0(x - x_0) \ .$$

Where does this line intersect the x-axis?? [Be CAREFUL. There is a trap.]

10. (5 pts.) On the back of Page 3 of 4, give a complete ε - δ proof that $\lim_{x \to -3}(7x - 9) = -30$ .

10 Point Bonus: Suppose $x_0 > 0$. Give an ε - δ proof that $\lim_{x \to x_0} x^{1/2} = (x_0)^{1/2}$ on the back of Page 2 of 4.