
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Be careful. Remember this: "=" denotes "equals" , "⇨" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

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1. (10 pts.) (a) Using a complete sentence and appropriate notation, provide the precise mathematical definition of continuity of a function f(x) at a point x = a.

(b) Explicitly using the definition of continuity, find all values for the constant c, if possible, that will make the function f(x) defined below continuous at x = 0 . Suppose

$$f(x) = \begin{cases} c^2 \cdot \cos(x/2) & , \quad x \geq 0 \\ \sin(25x)/x & , \quad x < 0. \end{cases}$$

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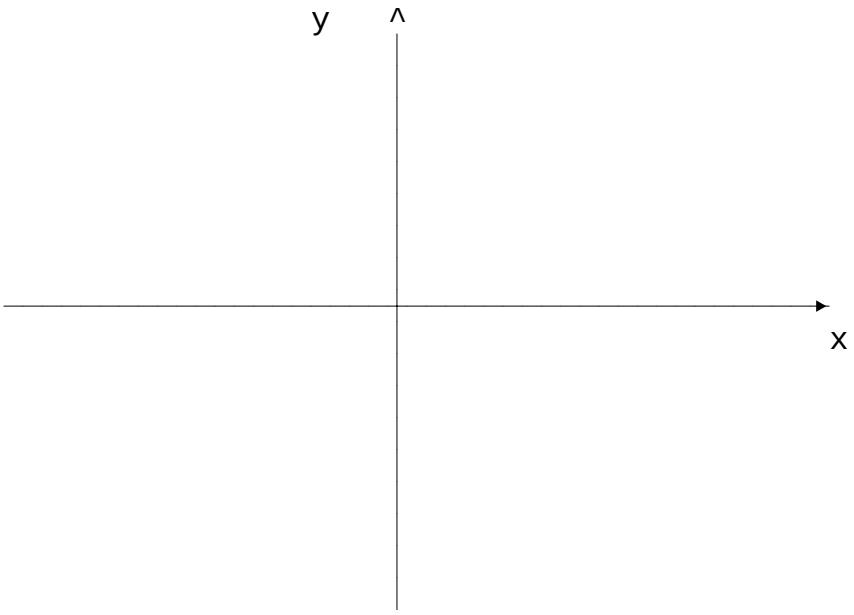
2. (15 pts.) (a) Using complete sentences and appropriate notation, provide the precise mathematical definition for the derivative, f′(x), of a function f(x).

(b) Using only the definition of the derivative as a limit, show all steps of the computation of f′(x) when $f(x) = (2x + 1)^{1/2}$.

3. (10 pts.) Locate and determine the maximum and minimum values of the function $f(x) = x + 4x^{-1}$ on the interval $[-4, -1]$. What magic theorem allows you to conclude that $f(x)$ has a maximum and minimum even before you attempt to locate them? Why??

4. (15 pts.) (a) Sketch the graph of the function $f(x) = |x|^3 + x^3$. [Hint: First write f in a piecewise-defined form.] (b) Find the derivative for $x > 0$ and for $x < 0$. [These are two separate cases.] (c) Show the computation of the one-sided derivatives at $x = 0$. What can you conclude from this??



5. (25 pts.) Obtain the derivative of each of the following functions. You may use any of the rules of differentiation at your disposal. Do not simplify the algebra. [5 pts./part]

(a) $f(x) = 7x^{-3} - 3x^5 + 20 \cdot x^{4/5}$

$f'(x) =$

(b) $g(x) = (x^5 - 7x^{-1})(25x^2 + 6x^{-5})$

$g'(x) =$

(c) $h(t) = \dfrac{3t^2 + 7t}{2t^4 - t^3}$

$h'(t) =$

(d) $L(x) = (x^4 + x^{-2})^{8/5}$

$L'(x) =$

(e) $y = (x^2 - 2x^{-2})^{25}(10x^{3/2} + 10x)^5$

$\dfrac{dy}{dx} =$

6. (10 pts.) (a) Using complete sentences and appropriate notation, state the theorem that is concerned with the intermediate value property of continuous functions.

(b) Apply the theorem concerning the intermediate value property of continuous functions to show that the given equation has a solution in the given interval.

$$x^5 - 6x^3 + 3 = 0 \text{ on } [-1,1]$$

Explain completely. Deal with all the magical hypotheses.

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7. (5 pts.) Give an example of a continuous function f(x) which has a point c in its domain with $f'(c) = 0$, but such that f(c) is not a local extreme value.

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8. (10 pts.) Find all points in the domain of

$$f(x) = x - 3x^{1/3}$$

where the tangent line is either horizontal or vertical, and clearly say which points have horizontal tangent lines and which have vertical tangent lines.