Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

1. (5 pts.) Use logarithmic differentiation to find dy/dx when $y = x^{\sec(x)}$. **Label your expressions correctly or else.**

2. (5 pts.) Use a linear approximation L(x) to an appropriate function f(x), with an appropriate value of a, to estimate the value of $80^{3/4}$.

3. (5 pts.) Write dy in terms of x and dx when $y = \sin(2x)e^{-3x}$.

dy =

4. (5 pts.) Find all points in the interval $[0,2\pi]$ where the graph of $f(x) = \sqrt{2}x + 2\cos(x)$ has a horizontal tangent line.

5. (5 pts.) Find the function f(x) that satisfies the following two equations: $f'(x) = \frac{3}{2}x^{1/2}$ *and* $f(1) = 10$.

6. (10 pts.) (a) Using implicit differentiation, compute dy/dx and when $x^4 + y^4 = 17$. **Label your expressions correctly or else.**

(b) Obtain an equation for the line tangent to the graph of $x^4 + y^4 = 17$ at the point (1,-2).

7. (10 pts.) Apply the first derivative test to classify the critical points of the function $f(x) = x^2e^x$. Reveal all the details of your analysis.

8. (5 pts) Rolle's Theorem states that if f(x) is continuous on [a,b] with f(a) = f(b) = 0 and differentiable on (a,b), then there is a number c in (a,b) such that f′(c) = 0. **Give an example of a function f(x) defined on [-1,1] with f differentiable on (-1,1) and f(-1) = f(1) = 0 but such that there is no number c in (-1,1) with f′(c) = 0.** [Hint: Which hypothesis above must you violate??]

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9. (10 pts.) (a) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

(b) Show how to use the Mean Value Theorem to prove the following: Suppose that a and b are real numbers with $a < b$. If f is a function that is continuous on a closed interval $[a,b]$ and is differentiable on the open interval (a,b) with $f'(x) < 0$ every x in (a,b), then f is a decreasing function on $[a,b]$.

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10. (10 pts.) A circular oil slick of uniform thickness is caused by a spill of 1 cubic meter of oil. The thickness of the slick is decreasing at the rate of .001 meter/hour. At what rate is the radius increasing when the radius is 8 meters. [Hint: The volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Obviously the same units must be used.]

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11. (5 pts.) Can there be two numbers x_1 and x_2 in the interval $[-1,1]$ where $\sin(x) = 3x - 1$?? Explain briefly. [Hint: Consider the function $f(x) = \sin(x) - (3x - 1)$ on $[-1,1]$.]

12. (10 pts.) Use the first derivative to determine the open intervals where the polynomial function $f(x) = x^4 - 2x^2$ is increasing and those intervals where f is decreasing. Be specific.

13. (10 pts.) What point on the line $2x + y = 24$ is nearest the origin, the point (0,0)?? [Hint: The square of the distance between points in the plane is easier to deal with than the distance and can tell you which point is closest.]

14. (5 points) Would you know a derivative if it hit you with a two by four? What is the value of the following limit??

$$\lim_{h \to 0} \frac{\cos(\frac{\pi}{2}+h) - \cos(\frac{\pi}{2})}{h} =$$