
Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page.*

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1. (10 pts.) Compute the first derivative of each of the following functions.

(a) $f(x) = \cos^{-1}(x)$ $f'(x) =$

(b) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(c) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(d) $f(x) = \cot^{-1}(x)$ $f'(x) =$

(e) $f(x) = \csc^{-1}(x)$ $f'(x) =$

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2. (10 pts.) Pretend y is a function of x. Using implicit differentiation, compute dy/dx and d^2y/dx^2 when $y^3 + x^2 + x = 5$. **Label your expressions correctly or else.**

3. (10 pts.) (a) Find all the critical points of the function

$$f(x) = 3(x^2 - 4x)^{1/3}$$

(b) Apply the second derivative test at each critical point, c, where f′(c) = 0, and draw an appropriate conclusion.

4. (10 pts.) Evaluate each of the following anti-derivatives.

(a)

$$\int 10x^9 + \frac{3}{x} - \frac{8}{x^5} + 6\sin(3x)dx =$$

(b)

$$\int 3e^x - \frac{x^4 + x^3 + 1}{x^2} - \frac{1}{(1-x^2)^{1/2}}\ dx =$$

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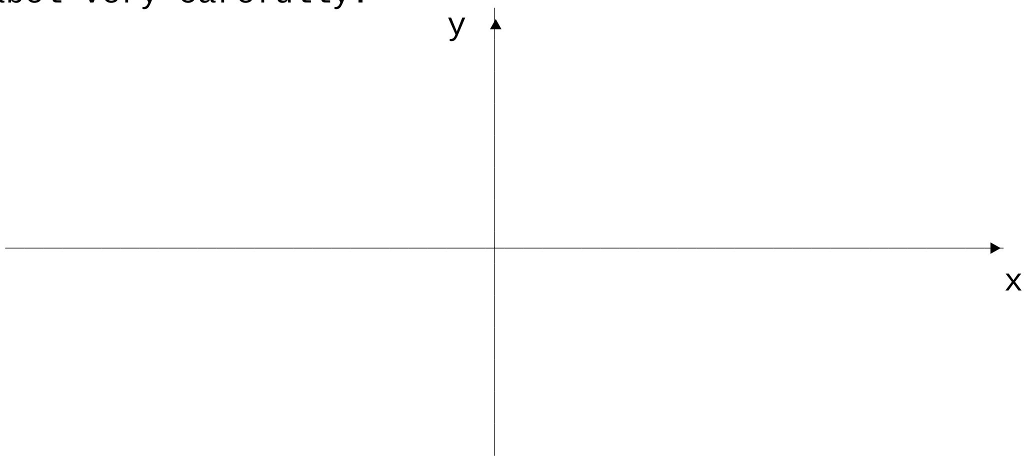
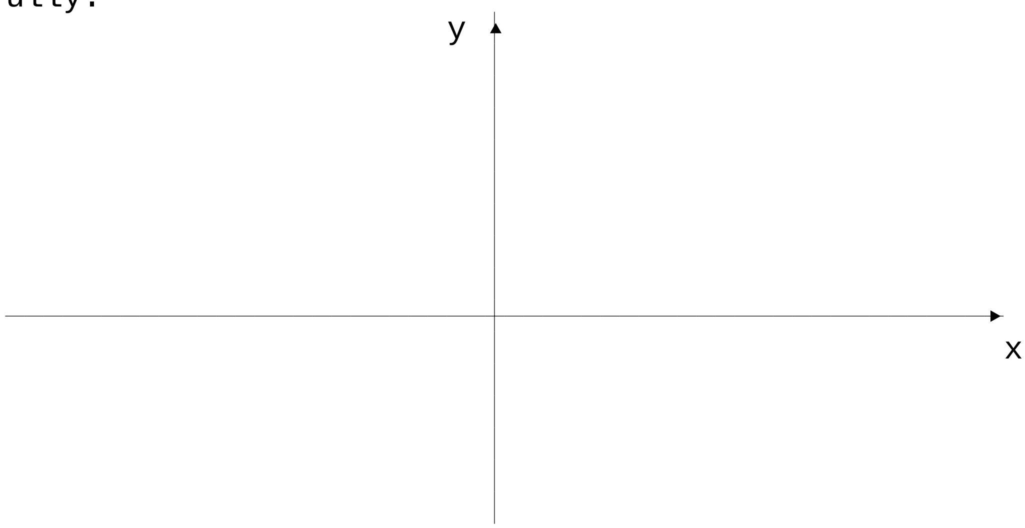
5. (10 pts.) (a) Carefully sketch the graph of $y = \tan^{-1}(x)$ below. Label very carefully.



(b) Carefully sketch the graph of $y = \cos^{-1}(x)$ below. Label very carefully.



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6. (10 pts.) (a) (5 pts.) Find a function g(x) so that g satisfies the following equation:

$$\int g(x)\, dx = \sec^2(x) + e^{2x} + 2x^5 + C$$

$g(x) =$

(b) Solve the following initial value problem:

$$\frac{dy}{dx} = 6e^{2x} \; ; \; y(0) = 10.$$

9. (20 pts.) Limits to lament. Evaluate each of the following limits. If a limit fails to exist, say how as specifically as possible. For some of these, L'Hopital's Rule may prove useful.

(a)

$$\lim_{x \to 0} \frac{\sin(4x)}{\tan(5x)} =$$

(b)

$$\lim_{x \to \infty} \frac{2x^3 - 1}{5x^2 + 3x^3} =$$

(c)

$$\lim_{x \to \infty} \left((x^2 + 2x)^{1/2} - (x^2 - 2x)^{1/2}\right) =$$

(d)

$$\lim_{x \to \pi/2} \frac{8x - 4\pi}{\tan(2x)} =$$

(e)

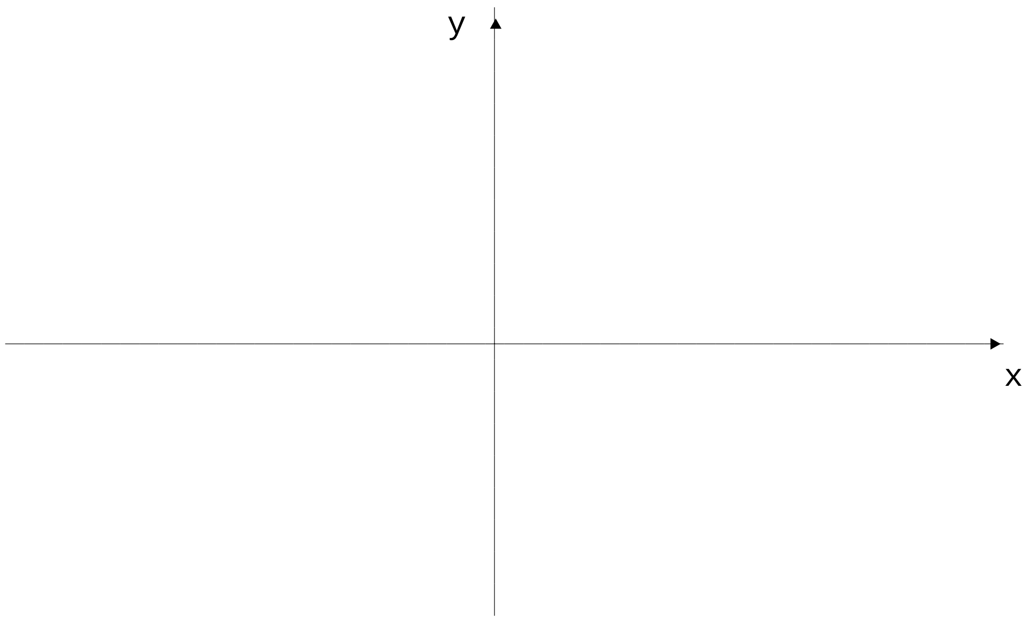
$$\lim_{x \to 0} \frac{1}{x} \ln\left(\frac{4x + 8}{7x + 8}\right) =$$

11. (15 pts.) Very carefully sketch the function

$$f(x) = 3x^4 - 4x^3.$$

Do this only after you have completely analyzed its behavior.

Analysis:



12. (5 pts.) A particle moves along the x-axis with the acceleration function $a(t) = 12t - 4$, initial position $x(0) = 0$, and initial velocity $v(0) = -10$. Find the particle's position function $x(t)$.

Silly Ten Point Bonus: Prove

$$\arctan(x) + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$$

whenever $x > 0$. Tell me where your work is, for there isn't room enough here.