Student Number: **Exam Number:**

Read Me First: *Show all essential work very neatly. Use correct notation when presenting your computations and arguments. Write using complete sentences. Remember this: "=" denotes "equals" , "⇒" denotes "implies" , and "⇔" denotes "is equivalent to". Do not "box" your answers. Communicate. Show me all the magic on the page. Eschew obfuscation.*

1. (36 pts.) Provide the first derivative for each of the following functions. These are not differentials.
3 points/part.

(a) $f(x) = \sin(x)$ $f'(x) =$

(b) $f(x) = \sin^{-1}(x)$ $f'(x) =$

(c) $f(x) = \sec^{-1}(x)$ $f'(x) =$

(d) $f(x) = \sec(x)$ $f'(x) =$

(e) $f(x) = \tan^{-1}(x)$ $f'(x) =$

(f) $f(x) = \tan(x)$ $f'(x) =$

(g) $f(x) = \log_{10}(x)$ $f'(x) =$

(h) $f(x) = 10^x$ $f'(x) =$

(i) $f(x) = \csc^{-1}(x)$ $f'(x) =$

(j) $f(x) = \csc(x)$ $f'(x) =$

(k) $f(x) = \ln(x)$ $f'(x) =$

(l) $f(x) = e^x$ $f'(x) =$

2. (36 pts.) Provide each of the following antiderivatives. **Do not forget the arbitrary constant.** 3 points/part.

(a) $\int \sec(x)\tan(x)\, dx$ =

(b) $\int \frac{1}{1+x^2}\, dx$ =

(c) $\int \frac{1}{x}\, dx$ =

(d) $\int \sin(x)\, dx$ =

(e) $\int \frac{1}{(1-x^2)^{1/2}}\, dx$ =

(f) $\int x^{10}\, dx$ =

(g) $\int 10^x\, dx$ =

(h) $\int \frac{1}{|x|(x^2-1)^{1/2}}\, dx$ =

(i) $\int \sec^2(x) dx$ =

(j) $\int \cos(x)\, dx$ =

(k) $\int csc^2(x)\, dx$ =

(l) $\int \ln(e^{55})\, dx$ =

3. (12 pts.) Evaluate each of the following easy limits. If a limit fails to exit, say how as precisely as possible. 1 point/part.

(a) $\lim_{x \to \infty} e^x =$

(b) $\lim_{x \to \ln(\pi)} e^x =$

(c) $\lim_{x \to -\infty} e^x =$

(d) $\lim_{x \to \pi/2} \frac{\sin(x)}{x} =$

(e) $\lim_{x \to 0} \frac{\sin(x)}{x} =$

(f) $\lim_{x \to \infty} \frac{\sin(x)}{x} =$

(g) $\lim_{x \to -\infty} \tan^{-1}(x) =$

(h) $\lim_{x \to -(3)^{1/2}} \tan^{-1}(x) =$

(i) $\lim_{x \to \infty} \tan^{-1}(x) =$

(j) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x =$

(k) $\lim_{x \to \infty} \ln\left(1 + \frac{1}{x}\right) =$

(l) $\lim_{x \to 0^+} \ln(x) =$

4. (24 pts.) Compute the derivatives of the following functions. You may use any of the rules of differentiation that are at your disposal. Do not attempt to simplify the algebra in your answers. 6 pts./part

(a) $f(x) = \sin(3x^2)e^{\pi x}$ $f'(x) =$

(b) $g(x) = \frac{\ln(4x)}{x^5}$ $g'(x) =$

(c) $h(t) = \ln(\tan^{-1}(x^2 - 1))$ $h'(t) =$

(d) $y = \sin(\sin^{-1}(x)) + (e^{\ln(x)})^2 + \tan^{-1}(45)$

$\frac{dy}{dx} =$

5. (9 pts.) Find the maximum and minimum values of the function $f(x) = x^{1/3}(8 - x)$ and where the extrema occur on the interval [-1, 8].

6. (9 pts.) Find a formula for the function g(x) which satisfies the following initial value problem: $g'(x) = -3\sin(x) + 2x$; $g(0) = 10$.

7. (9 pts.) Use logarithmic differentiation to differentiate

$$f(x) = x^{\sec(x)}$$

for $x \in (0, \pi/2)$.

8. (9 pts.) A spherical weather balloon is inflated so that its volume is increasing at a constant rate of 4 cubic feet per minute. How fast is the diameter of the balloon changing when the radius is 7 ft.?
[In case you forgot, $V = (4/3)\pi r^3$.]

9. (12 pts.) Evaluate each of the following limits. L'Hopital may help. Squeezing may help. Nothing may help.

(a) $\lim_{x \to 0} \dfrac{8 - 8\cos(x)}{e^{x} + e^{-x} - 2} =$

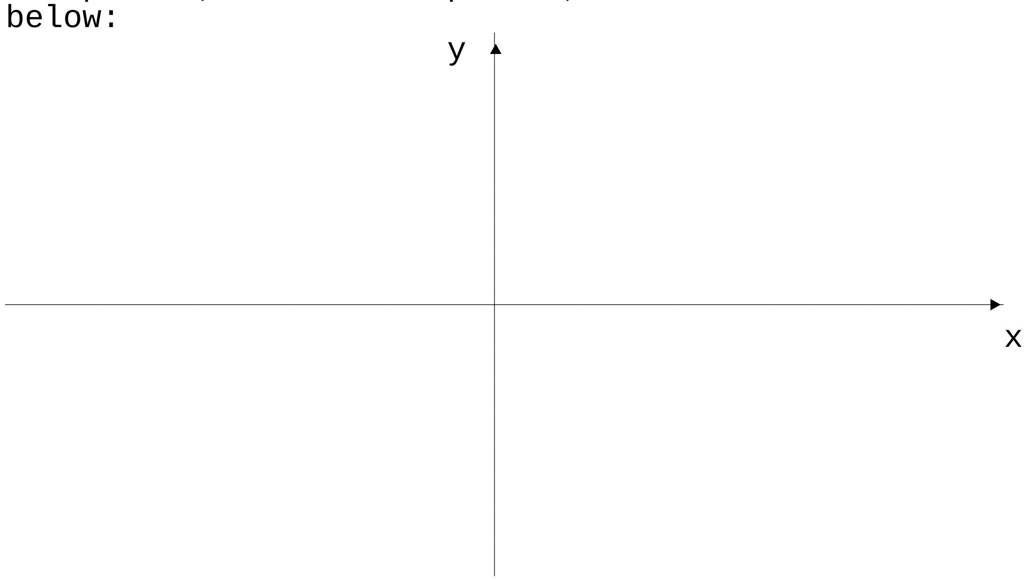
(b) $\lim_{h \to \infty} h\left[\sec\left(\dfrac{\pi}{3} + \dfrac{1}{h}\right) - 2\right] =$

10. (12 pts.) Evaluate each of the following antiderivatives.

(a) $\int x e^{x^2 - 1} + \dfrac{\ln^2(x)}{x}\, dx =$

(b) $\int \dfrac{1}{9 + x^2} - \dfrac{2x}{(1 - x^4)^{1/2}} dx$

11. (12 pts.)
Let $f(x) = 3x^4 - 4x^3$. Analyze f′ and f″ and how f behaves as x ⟶ ±∞. Plot critical points, inflection points, and zeros. Then sketch the graph carefully below:



Analysis:

12. (8 pts.) (a) Using a couple of sentences, precisely provide the mathematical definition of

$$\lim_{x \to a} f(x) = L$$

in terms of epsilons and deltas.

(b) Using complete sentences and appropriate notation, provide the precise mathematical definitions for continuity of a function f(x) at a point x = a.

(c) Using complete sentences and appropriate notation, state the Intermediate Value Theorem.

(d) State the Mean Value Theorem of Differential Calculus. Use a complete sentence and appropriate notation.

13. (12 pts.) Using only the definition of limit at a point in terms of epsilons and deltas, give a proof that

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2.$$

Hint: This is really linear, really.

Silly 20 point Bonus Problem: (a) Use the Mean Value Theorem to prove that if $x \geq 1$, then $\ln(x) < x$. (b) Show how to use the Mean Value Theorem to estimate the value of $(24)^{1/2}$. Does this allow you to obtain a better estimate of $6^{1/2}$ than an application of M.V.T. on the interval [4,6]??